

XIII. On the laws of polarisation and double refraction in regularly crystallized bodies. By David Brewster, LL.D. F. R. S. Lond. and Edin. In a letter to the Right Hon. Sir Joseph Banks, Bart. G. C. B. P. R. S.

Read January 15th, 1818.

Venlaw, June 1, 1817.

DEAR SIR,

IN the different memoirs which you have done me the honour of submitting to the Royal Society, I have considered principally those branches of the polarisation of light which relate to the superficial action, or the superinduced properties of uncryallized bodies. In the course of these enquiries, my attention was frequently directed to the phenomena of regular crystals; but from the difficulty of procuring proper specimens, and the extreme perplexity of the subject, it was not till lately that I succeeded in reducing under a general principle all the complex appearances which result from the combined action of more than one axis of double refraction.

Before I proceed to trace the steps which have conducted me to this general law, I must entreat the indulgence of the Society, while I attempt to give a brief and rapid view of the present state of our knowledge respecting the laws of double refraction. They will thus be able to appreciate more correctly the relative value of those successive generalisations by which this subject has been raised to one of the most interesting departments of physical science.

Dr. THOMAS YOUNG was the first person who directed the attention of philosophers to the perfect agreement between the beautiful theory of HUYGENS, and the double refraction of light as exhibited in calcareous spar. (see Phil. Trans. 1802, p. 45). The experiments of Dr. WOLLASTON afforded additional evidence of its exactness, and the more numerous and diversified observations of MALUS raised it to the rank of a general principle, which represented all the phenomena in the most accurate manner. About the same time, M. LAPLACE attempted to refer the deviation of the extraordinary ray to the action of those attractive and repulsive forces by which the ordinary refraction and reflection of light are produced. In this theory the aberration of the extraordinary ray is explained by a repulsive force emanating from the short diagonal of the rhomb of calcareous spar, or the axis of extraordinary refraction ; and it is shown that the difference of the squares of the velocities of the ordinary and extraordinary ray is proportional to the square of the sine of the angle which this last ray forms with the axis ; and that this difference represents that of the action of the crystal upon the two kinds of rays. LAPLACE then demonstrates that the principle of FERMAT, and the principle of least action lead to the law of HUYGENS, provided that in the principle of FERMAT the radius of the ellipsoid is taken to represent the velocity, while in the principle of least action it is made to represent the time in which light describes a certain space taken for unity.

This fine theory will no doubt be received by philosophers as a new proof of the high genius of its author ; but while it is

thus received, we ought carefully to consider whether the HUYGENIAN law, upon which it rests, is the universal law of double refraction, or merely an elegant and correct expression of the individual phenomena of calcareous spar. Because this law represents with accuracy the action of a single crystal, is it thence to be presumed that all the other transparent crystals of the mineral kingdom possess the same structure, and exhibit the same series of appearances as calcareous spar? We venture to say that such a conclusion is scarcely possible in the present state of science; and that if it should happen to have been rashly drawn, it could not be deliberately supported by any one who is imbued with the cautious spirit of the inductive philosophy.

It will be replied, however, to this question, that MALUS has examined, with the utmost care, the properties of *quartz*, *arragonite*, and *sulphate of barytes*; that he has demonstrated the identity of their action with that of calcareous spar; and that the extension of the law of HUGENS to other crystallized bodies can no longer be doubtful. This reply would have possessed considerable weight, had the alledged identity of action been satisfactorily established; but the experiments of MALUS are decidedly erroneous, for out of the *three* crystals, the identity of whose action with that of calcareous spar he is supposed to have demonstrated, no fewer than *two* have more than one axis of double refraction.

Here then is a difficulty of an unexpected kind: for if the experiments of MALUS on these crystals are set aside as incorrect, what confidence can we place in his observations on calcareous spar, upon which the truth of the HUYGENIAN law is founded? If nitre and arragonite, both of which have a

powerful double refraction, had, in place of calcareous spar, been put into the hands of HUYGENS, Dr. WOLLASTON, or MALUS, or of any other philosopher, however skilful, it is demonstrable that their measurements would have accorded with the theory of spheroidal undulations. LAPLACE would have connected this theory with the principles of mechanics, and it would have been universally received as a rigorously physical law. Yet after all this display of experimental and mathematical genius, the whole investigation would have turned out a delusion; for it can be shown, by decisive experiments, that both *nitre* and *arragonite* have two axes of double refraction; and that the aberration of the extraordinary ray cannot be explained by a single ellipsoid.

From these observations, therefore, we conceive it to be proved, not only that the HUYGENIAN law remains undemonstrated as the general law of double refraction; but that it remains undemonstrated as a correct expression of the individual phenomena of calcareous spar.

How then, it will be asked, are we to determine the law of extraordinary refraction, if the best experiments of our most eminent philosophers are set aside as insufficient? This can only be done in two ways, either by the discovery of new methods of magnifying and measuring with great exactness the deviation of the extraordinary ray, when the light passes near the axis of a crystal; or what is more practicable and correct, by examining the appearances which are exhibited by transmitting polarised light along the real or apparent axes of double refraction.

In the year 1811, M. ARAGO discovered, that when polarised light was transmitted through thin plates of sulphate

of lime and mica, and afterwards analysed with a prism of calcareous spar, they exhibited the most beautiful complementary colours. Without knowing what had been done by M. ARAGO, the same discovery was made by me in 1812: but though the general fact which each of us discovered was the same, yet I had the good fortune, by a peculiar mode of observation, to examine the phenomena seen along the real or apparent axes of crystals, and have thus been led to the results contained in the following paper,—results which, as will be seen in the sequel, could not possibly have been obtained by the mode of observation employed by M. ARAGO, and afterwards by M. BIOT.

The last of these philosophers has examined this class of phenomena with singular diligence and ingenuity. By a series of fine experiments, he has established many fundamental points in the science, and has associated his name with this branch of physics, as one of its most distinguished and successful cultivators. The crystals which M. BIOT examined were

Calcareous spar.	Tourmaline.	Sulphate of lime.
Rock crystal.	Feld-spar.	Sulphate of barytes.
Beryl.	Arragonite.	Sulphate of strontian.
Phosphate of lime.	Topaz.	Mica.

But his experiments for investigating the laws of moveable polarisation were principally made with *sulphate of lime*, on account of its perfect transparency, and the facility with which it can be separated into thin plates. The general result of these experiments is, that all the preceding crystals, with the single exception of certain specimens of mica, have

only one axis of double refraction from which the polarising force emanates ; that all crystals are divisible into two classes, namely, attractive and repulsive ; and that the laws investigated for sulphate of lime, abstracting the effects of certain secondary forces, are the general laws of polarisation for all other crystals.*

This view of the laws of polarisation, though deduced from ingenious investigations, and by methods which exhibit the finest talents for physical research, is incompatible with the experiments and observations contained in the following paper. No fewer than *seven* out of the *twelve* minerals employed by M. BIOT, have two or more axes of double refraction. Sulphate of lime itself belongs to this number ; and all the irregularities of its action, which M. BIOT has represented by empirical formulæ, are the legitimate and calculable results of two rectangular axes. The division of crystals into attractive and repulsive, and the reference of the phenomena of mica to two repulsive axes, whose intensities are as 100 to 677, will also be found to be entirely hypothetical.

From this slight sketch of the present state of the science, it will be readily seen that the process of generalisation, however ingeniously it has been managed, has been carried on too rapidly, and has far outrun the progress of observation and experiment. In attempting, therefore, to establish new and more general laws, I began my researches by laying a broad foundation of facts. One hundred and sixty-five crystals were subjected to examination. In 165 of these, I have observed the property of double refraction. In about 100, I

* See BIOT's *Traité de Physique*, tom. iv. p. 377.

have been able to ascertain whether they have one or more axes; and by examining and measuring the tints which they exhibit at various angular distances from the axes whence the forces emanate, I have been led to a general principle which embraces all the phenomena, and extends to the most complex, as well as to the most simple developement of the polarising forces. This general principle is in no respect an empirical expression of the facts which it represents, nor is it supported by any empirical data. Founded on the principles of mechanics, it is a law rigorously physical, and enables us to calculate all the tints, &c. of the coloured rings, and all the phenomena of double refraction, with as much accuracy as we can compute the motions and positions of the heavenly bodies.

In the course of researches embracing the examination of such a great number of crystals, it was natural to expect that many new facts would present themselves that were but slightly connected with the general object of investigation. The phenomena of this kind, which I have had the good fortune to observe, comprehend several new properties of light, and of crystallized bodies, which I shall lay before the Society in a series of separate communications.

SECTION I. *On the crystals which produce double refraction.*

The property of double refraction was observed by the Abbé HAUY in *twenty* crystallized substances.* MALUS has given a list of *nineteen* doubly refracting crystals, embracing

* *Traité de Mineralogie*, tom. i. p. 271, 272, Paris, 1801.

all the bodies in HAUY's table, except topaz, which he has accidentally omitted.*

The faculty of depolarisation, which I have explained in a former paper,† has been considered as a sufficient indication of the existence of two separate images; and upon this principle it has been stated, that all crystals have the property of double refraction, whose primitive form is neither the cube nor the regular octohedron.‡ This statement, however, is by no means correct, for the rhomboido-dodecahedral crystals ought also to have been excepted; and I have besides shown, by numerous experiments, that almost all the crystals which have these forms, exhibit an imperfect doubly refracting structure, which some of them possess to a very considerable degree. But admitting the statement to be unexceptionable, it could never have been used as a rule for determining whether a crystal refracts doubly or singly; for it is much more difficult to detect the primitive form of a crystal, than to examine, by direct experiments, its optical properties. Tungstate of lime, for example, would have been reckoned a crystal without double refraction, when HAUY believed its primitive form to be the cube, although it possesses this property in a very high degree.

In order, therefore, to determine whether crystals have the property of double or of single refraction, we must ascertain by direct observation, if they form two images, or possess the property of regular depolarisation. In this way I have obtained the results in the following table; but as the exist-

* *Theorie de la Double Refraction*, p. 96, Paris, 1810. † *Phil. Trans.* 1815, p. 27.

‡ *Edin. Transactions*, vol. viii. Part. I. BIOT's *Traité de Physique*, tom. iii. p. 325.

ence of two separate images is the only correct and infallible test of double refraction, I have employed it for the greater part of the substances in the table. When a crystal possesses the property of depolarising light, it only proves that it forms two pencils polarised in opposite planes; though there can be little doubt that one of them is subject to the extraordinary law of refraction.

Table of doubly refracting crystals.

Chromate of lead	Tourmaline
Carbonate of lead	Rubellite
Tungstate of lime	25 Quartz
Carbonate of potash	Agate
5 Nitrate of potash	Dichroite
Calcareous spar	Bitter spar
Arragonite	Apatite
Zircon	30 Idocrase
Sulphur	Mica
10 Acetate of lead	Lepidolite
Acetate of nickel	Talc
Ruby	Indurated talc
Sapphire	35 Chlorite
Corundum	Kyanite
15 Beryl	Stilbite
Emerald	Cubizite
Cymophane	Apophyllite
Peridot	40 Prehnite
Epidote	Hydrate of magnesia
20 Euclase	— barytes
Topaz	— strontites
Pycnite	Feldspar

45	Mellite	Spodumene
	Realgar	Tabular spar
	Native orpiment	Benzoic acid
	Ruby silver	Oxalic acid
	Actynolite	
50	Anhydrite	80 Citric acid
	Litharge	Tartaric acid
	Cinnabar	Boracic acid
	Tincal, or native borax	Phosphoric acid
	Borate of soda	Succinic acid
55	Mesotype	85 Chromic acid
	Sphene	Cryolite
	Harmotome	Benzoate of ammonia
	Macle	Sulphate of cobalt
	Nepheline	— — — — lead
60	Augite	90 — — — — iron
	Datolit	— — — — zinc
	Wavellite	— — — — copper
	Calamine	— — — — copper and iron
	Anthophyllite	— — — — nickel
65	Laumonite	95 — — — — barytes
	Titanite	— — — — strontites
	Asbestos	— — — — lime
	Serpentine	— — — — magnesia
	Steatite	— — — — potash
70	Diopside	100 — — — — soda
	Staurolite	— — — — ammonia
	Diallage	— — — — magnesia and
	Hyacinth	— — — — soda
	Tremolite	Carbonate of copper
75	Semiopal	— — — — barytes
		105 — — — — strontites

Carbonate of soda	Phosphate of iron
— ammonia	135 — copper
— copper	— lead
Muriate of mercury	— soda
110 — gold	— sub of potash
— silver	Arseniate of lead
— copper	140 — iron
— iron	— copper
— lead	— potash
115 — magnesia	Tartrate of potash
— lime	— potash and soda
— barytes	145 — super of potash
— strontites	— potash and an-
Nitrate of silver	timony
120 — copper	Oxalate of ammonia
— zinc	Superoxalate of potash
— bismuth	Prussiate of potash
— mercury	150 Hyperoxymuriate of
— ammonia	potash
125 — lime	Oxymuriate of mercury
— strontites	Calomel
— soda	Mother of pearl
— magnesia and	Ice
ammonia	155 Camphor
Acetate super of copper	Sugar
130 — super of copper	Crystallized Cheltenham
and lime	salts
— zinc	Murio-sulphate of iron
— soda	and magnesia
— potash	Emerald copper

160	Muriate of lead	manganese
	Sulphate of ammonia and	— alumine
	magnesia	Specular iron
	— red oxide of 165	Pargasite

SECTION II. *On crystals with one apparent axis of polarisation.*

Having thus distinguished the crystals which possess double refraction, from those which are destitute of this property, we shall now proceed to determine their optical structure, or to ascertain whether the forces which act upon the extraordinary ray emanate from one or more axes.

If we transmit polarised light through any of the parallel surfaces of the primitive hexaedral prism of a crystal of beryl, and analyse the emergent pencil by a prism of calcareous spar, having its principal section placed either parallel or perpendicular to the plane of primitive polarisation, it will be found that, when the axes of the prism of beryl is inclined 45° to the plane of primitive polarisation, the vanished image is restored. If the thickness of the crystal exceeds 0.035 of an inch, the restored light will be nearly white.

But if the polarised light is transmitted along the axis of the prism of beryl, there will be seen a series of beautiful circular concentric rings whose centre is the intersection of the arms of a dark rectangular cross, and containing all the tints in NEWTON's table of the colours of thin plates. These rings, which are shown in Plate xv. fig. 1, increase in diameter as the length of the prism is diminished; but they may be distinctly seen, even in the longest prism of beryl. If we now examine the double refraction of beryl by transmitting

the common light through two surfaces very much inclined, it will be seen that the separation of the two images is a *maximum*, when the transmitted ray is perpendicular to the axis of the prism, and that the two images gradually coalesce into one, as the transmitted ray becomes parallel to the axis. The axis of the prism, therefore, is the apparent axis of double refraction, and likewise the axis of the system of rings shown in fig. 1. Hence we conclude that beryl has one apparent axis of extraordinary refraction, and one apparent axis of polarisation which are coincident. The same property of producing the rings in fig. 1, I have discovered in the following crystals.

A list of crystals with one apparent axis of double refraction and polarisation.

Beryl, emerald	Muriate of strontian
Calcareous spar, bitter spar	15 Apophyllite
Sapphire, ruby	Nitrate of soda
Rubellite, tourmaline	Subphosphate of potash
5 Phosphate of lime	Sulphate of nickel (certain specimens of)
Hydrate of magnesia	Zircon
— strontites	20 Quartz
Idocrase	Ice
Mellite	Super acetate of copper and lime
10 Nepheline	Sulphate of potash (certain specimens of)
Arseniate of copper	
Potash	
Muriate of lime	

As several of the crystals contained in the preceding list are not capable of being cut into transparent prisms with large refracting angles, it is impossible to ascertain the relation between the force of double refraction, and the polarising

force, by the action of which the coloured rings are produced; but, in all those crystals upon which this experiment can be made, I have invariably found that the two axes from which these forces emanate, are coincident, and that the force which produces the deviation of the extraordinary ray increases and diminishes with the polarising force which produces the coloured rings.*

In proceeding to examine the nature and properties of the coloured rings produced by the different crystals in the table, we shall first consider the effect produced by plates of different thicknesses. If we take a rhomboid, of calcareous spar, whose principal section is A B C D, fig. 2, and cement upon its two surfaces AB,CD, two prisms of flint glass, having their sides perpendicular to the principal section, and their refracting angles EBF, GDH, a little greater than FBb which is $45^{\circ} 23' 26''$; and if polarised light is incident, perpendicularly upon the surfaces BE,DG, it will be transmitted parallel to Bb,† and will exhibit in the most beautiful manner the system of coloured rings shown in

* M. BIOT was the first who deduced this conclusion for crystals with one axis from experiments on calcareous spar and rock crystal; but it could not be considered as a general fact when drawn from such a small number of crystals. It will be seen in the sequel of this paper, that I have extended the result to crystals with two axes, and have thus established it as a general principle.

† This mode of exhibiting the coloured rings, is greatly superior to the mode employed by BIOT, of grinding down the solid angles at B and D. Beside the advantage of procuring the greatest possible thickness from a given rhomboid, we preserve the polish of its natural faces, and the thickness of crystal through which the polarised ray passes can be calculated with the utmost accuracy from the thickness Bb of the plate. This method is also peculiarly favourable for showing the influence of pressure upon the polarising structure of the crystal, and for various other experiments on the coloured rings.

fig. 1.* Let the rhomboid ABCD be now cut into two plates, by any line MN, and let the rings, produced by each of the plates, be examined separately in the way already described: it will then be found that the squares of the diameters of the rings are in every case proportional to the numbers which represent the corresponding tints in NEWTON's table; and that the squares of the diameters of similar rings, as produced by plates of different thicknesses, are reciprocally proportional to the square roots of these thicknesses. These two results, which were first obtained by M. BIOT, lead to the general conclusion that the tints produced at different inclinations to the axis of the crystal, are to one another, as the square of the sine of the angle which the polarised ray forms with that axis. This law of the tints for crystals with one axis, was deduced by M. BIOT from the action of rock crystal, as well as calcareous spar; and also from the phenomena of certain specimens of mica which he supposes to have one axis. I have found it perfectly correct in all the other crystals contained in the table; and it may therefore be considered as a general law, which we may apply with confidence in our future researches.

* The system of coloured rings produced by one axis of double refraction, and the still more beautiful and complicated system produced by two axes, were discovered by me in the year 1813. I observed the former in *beryl*, *emerald*, *ruby*, &c. and the latter in *topaz*, *mica*, and a great variety of other minerals. Dr. WOLLASTON was the first who detected the circular system of rings in *Iceland spar*, and they were shown to me by that eminent philosopher in July 1814. In a letter dated Dec. 3, 1815, M. BIOT announced to me, that he had then discovered the circular rings in *Iceland spar*, and it appears that the same observation was made by M. SEEBECK, in Dec. 1815; these dates, however, are nearly a year and a half posterior to that of Dr. WOLLASTON's experiment.

In order to apply it however with the utmost simplicity, I have found it very convenient to consider every crystal as cut into a sphere, one of whose diameters is the axis of double refraction, and to suppose that the polarised ray passes through the centre of the sphere. By this means we have no occasion to consider the refractive power of the crystal, or the difference of thickness arising from oblique transmission; for the polarised ray is always incident perpendicularly, and the thickness of the sphere is every where the same. If in this sphere, AB, fig. 3. is the axis of the system of rings, then Pp may be called the *diameter of no polarisation*, or the *apparent axis of double refraction and polarisation*; P and p the *poles of no polarisation*; COD, the *equator of maximum polarisation*, and EF the *isochromatic lines or curves of equal tint*. Now, since the tint varies as the square of the sine of the angle which the transmitted ray forms with the axis Pp , it will be a *maximum* in every point of the equator COD, and will be represented by $\text{Sin}^2. 90^\circ$. Hence if ϕ be the angle which any other diameter EO forms with the axis, the tint at E, and at every point of the parallel EF, will be represented by $\text{Sin.}^2 \phi$. By determining therefore experimentally the tint t , produced at any given thickness B, and at any inclination ϕ , the maximum tint T, for that thickness, will be $T = \frac{t}{\text{Sin.}^2 \phi}$ and the tint for any other thickness b will be $T = \frac{b}{B} \times \frac{t}{\text{Sin.}^2 \phi}$. It is obvious from these formulæ, that any given tint can be developed at any angle or distance from the axis, merely by varying the thickness of the crystal or the diameter of the sphere.

If it should be required to find the tints corresponding to any angle of incidence upon the natural faces of the crystal,

we have only to consider the position of our sphere within the crystal, and compute the inclination of the refracted ray to the axis. Thus in calcareous spar the axis Pp corresponds with the short diagonal AB of the primitive rhomb, as shown in fig. 3. In *beryl*, *zircon*, *sapphire*, *rubellite*, *rock crystal*, *nepheline*, *idocrase*, *muriate of lime*, *super acetate of copper and lime*, *muriate of strontian*, &c. it corresponds with the axis of the prism in which they generally crystallize; while in *hydrate of magnesia*, *arseniate of copper*, &c. it is perpendicular to the laminæ of which they are composed.

Although the systems of rings produced by all crystals with one axis, exhibit the same tints, and possess the same properties, yet those which are produced by the five crystals of the negative class, namely, *zircon*, *quartz*, *ice*, *super acetate of copper and lime*, and certain specimens of *sulphate of potash*, will be found to differ in one essential particular from the system produced by all the other crystals. If we take two rhomboids of calcareous spar and place the one symmetrically on the other, the system of rings which they exhibit will be exactly the same as would have been produced by one rhomboid whose thickness is equal to the sum of the thicknesses of the two which are combined. M. BIOT has deduced this result from theory, and attempted in vain to confirm it by experiment;* but from the peculiarity of the method of observing

* J'ai employé ainsi conjointement les deux plaques de spath d'Islande dont j'ai donné plus haut les épaisseurs; et j'ai eu en effet des anneaux bien plus petits que par une seule d'entre elles; mais la difficulté d'aligner également les axes de ces deux plaques m'a empêché de mettre dans les expériences l'exactitude nécessaire pour les mesurer. J'aurais voulu aussi combiner des cristaux attractifs avec des cristaux repulsifs, mais je n'en ai pas eu l'occasion. BIOT *Traité de Physique*, tom. iv.

the rings shown in fig. 2, I have been able to prove it by direct observation. The exact parallelism of the sides of the two rhomboids, when their surfaces are not perfectly flat, can easily be obtained by separating them with a piece of soft wax, and observing the perfect coincidence of the reflected images from the two adjacent surfaces; or by looking at the system of rings, and altering the position of the crystals till they become quite perfect. I have obtained a similar result by combining a plate of *beryl* with a plate of *calcareous spar*, for the system of rings will always be the same as would have been produced by two plates of beryl, one of which was the plate employed, and the other a plate which gave rings of the same size as the plate of calcareous spar.

But when we combine the system of rings produced by a crystal of *zircon*, &c. with the system produced by *calcareous spar*, a very different effect is produced. The system of rings instead of being diminished is increased, and is equal to the system which would have been produced by a thin plate of calcareous spar, whose thickness is equal to the difference of the thicknesses of the plate of calcareous spar employed, and another plate of calcareous spar, that would give rings of the same size as those given by the zircon, &c. alone. This result, which I succeeded after much labour in obtaining experimentally, will also be obtained by substituting ice in place of zircon. Quartz cannot be employed in these experiments, as the system of rings is never complete, on account of the secondary tints which M. BIOT discovered along its axis. If the plate of zircon, &c. gives a system of rings of the very same size as those of calcareous spar, the one system will be com-

pletely obliterated by the other, and the combined crystals would exhibit neither double refraction nor polarisation. Hence it follows that *zircon*, *quartz*, and *ice*, *superacetate of copper and lime*, and certain specimens of *sulphate of potash*, form a class separate from all the other crystals in our table ; and that the polarising force in the one class, is either of an opposite nature, or exerted in an opposite direction to the polarising force in the other class. This opposition in the action of crystals was first observed by M. BIOT, in plates cut obliquely to the axis ; and by this mode of observation he divided the crystals which he examined, into two classes, in the following manner :

REPULSIVE CLASS.	ATTRACTIVE CLASS.
Calcareous spar	Rock crystal
<i>Arragonite</i>	<i>Sulphate of lime</i>
Beryl	<i>Sulphate of barytes</i>
Tourmaline	<i>Topaz</i>

But since the crystals marked in italics have two or more axes of double refraction, and cannot, as we shall afterwards show, be ranked in either class, M. BIOT's list, when corrected, will stand thus :

REPULSIVE CLASS.	ATTRACTIVE CLASS.
Calcareous spar	Rock crystal
Beryl	
Tourmaline	

agreeing, in so far as it goes, with the more extended table which we have already given. Hence it is obvious that M. BIOT could not have made the experiment which he proposes to make, at the end of the passage just quoted; namely, to

combine the system of rings formed by the attractive crystals, with the system formed by the repulsive ones; for rock crystal being the only attractive substance of one axis with which he was acquainted, was quite unfit for this purpose, from the imperfection of the system of rings which it produces. The discovery of the optical structure of *zircon** has enabled me to perform the experiment with success.

This opposition of structure between *quartz* and the other four crystals with one axis, contained in M. BIOT's table, is ascribed by this eminent philosopher to an opposition in the forces which act upon the extraordinary ray; and he considers it as an established point, that in rock crystal, the extraordinary ray is attracted to the axis, while in the other crystals it is repelled from it. Hence he has concluded, that the phenomena may be represented in the first case by a prolate ellipsoid, and in the second case by an oblate ellipsoid as HUYGENS had already shown; and has thus been led to divide crystals into two great classes, *attractive* and *repulsive*. This division, however, is entirely hypothetical, in so far as the polarising forces is concerned, for it will afterwards be shown that the phenomena of rock crystal may be explained by a negative force emanating from two equal rectangular axes, or that the phenomena of the other class may be explained by a positive force in a similar manner; or if we prefer the analogies which the sciences of magnetism and

* The extreme difficulty which attends experiments of this kind, will be understood from the fact, that I cut more than fifteen plates out of a large piece of *zircon* without discovering its axis. By a singular accident, however, Mr. MORTON, jeweller, in Edinburgh, procured for me no fewer than sixty plates of *zircon* with parallel faces, and it was only in two of these that the system of rings was developed.

electricity present, we may explain the phenomena of both classes, by supposing each crystal to be endowed both with positive and negative axes.

In marking, therefore, this difference of action, I have employed the terms *positive* and *negative*, as denoting merely the *opposition*, and not the *nature* of the polarising forces.

Hitherto we have considered the system of coloured rings as produced only near the axes of crystals, or as capable of being developed at any distance from the axis, merely by diminishing the size of the sphere; but there are two modes by which the rings can be rendered visible at any distance from the axis, and with any thickness of crystals.

If we take a prism of flint glass, with a large refracting angle, and examine through it the system of rings, we shall find that instead of nine or ten rings, which are visible without the prism, we may reckon, by estimation, from 80 to 100 on that side of the axis towards which the refraction is made. This observation is analogous to that of NEWTON upon the rings formed by thin plates, and decidedly proves that the coloured tints are actually produced at distances from the axis where the phenomena of fixed polarisation are exhibited.

The other mode of developing the rings at any distance from the axis, and with any thickness of crystal, consists in crossing the tints with plates of *rock crystal* cut parallel to its axis of double refraction, or with laminæ of *sulphate of lime*. If the axis AB, Pl. XV. fig. 1, of a plate of rock crystal, having such a thickness as to produce no coloured tints, even at a considerable obliquity, is placed in the direction of a diameter of the rings given by calcareous spar, a new system of rings will be produced at A and B, having their centres at O. The

point where A,B, crosses the middle ring will be black, as the action of the calcareous spar is there exactly counterbalanced by the action of the rock crystal. The rings within CD, EF will be *positive*, in consequence of the action of the rock crystal predominating, and those without them will be *negative*, in consequence of the action of the calcareous spar predominating. If the plate of *rock crystal* or *sulphate of lime* produces the rings at *a*, *b*, instead of A,B, then in the other two quadrants, *c*, *d*, the tints will be those which are produced by the sum of the actions of the crystals, and the tints at *c* and *d* will be double of that which is produced at these points by the calcareous spar alone.*

One of the most interesting features of the system of coloured rings, is the black cross MN, PR, where the two arms are exactly at right angles to each other. If the plate which produces this system of rings is turned round its axis O, the rings and the black cross will preserve an invariable

* The developement of colours by the opposite action of two crystallized plates, which are not capable of producing them separately, was first effected by M. BIOT, before the discovery of the coloured rings, and by the aid of his divided apparatus ; but the preceding method of developing the rings by these opposite actions, is more general, and is so extremely simple, that it can be easily done by any person, merely by holding the crystals in his hands. The difficulties experienced by M. BIOT are thus stated by himself : " Mais telle est l'exactitude qu'il faut mettre à ces recherches que mes premières tentatives n'eurent aucun succès, parce que je m'étais borné à opérer le croisement à la main au lieu d'employer les appareils divisés qui m'avaient servi jusqu'alors. Car la grande énergie du spath d'Islande fait que si l'on s'écarte le moins du monde des positions indiquées par la théorie, on passe à la polarisation totale et les couleurs ne peuvent plus se developper. Cette cause faisait aussi qu'il était plus difficile de tomber précisément sur la proportion des épaisseurs de chaux sulfatée qui pouvait servir à compenser les lames de cristal d'Islande soumises à l'expérience."

position, one of the arms, MN, being always in the plane of primitive polarisation, and the other, PR, in a plane perpendicular to it. Hence it follows, that whenever a plane passing through the axis of the crystal, is either in the plane of primitive polarisation, or perpendicular to it, the extraordinary pencil or tint disappears. This result will be found correct, whatever be the direction in which the polarised ray traverses the crystal, and is therefore the general law of the disappearance of the extraordinary pencil for crystals with one axis.

SECTION III. *On crystals with two or more axes of polarisation.*

The complicated structure of mica was first discovered in 1812 by M. BIOT and myself, without any knowledge of each others observations. M. BIOT considered one kind of mica as the only mineral known to possess this compound structure, indicating the existence of two axes; whereas, so early as the beginning of 1813, I had found the same structure in *topaz*,* *nitre*, *sulphate of potash*, *tartrate of potash and soda*, *acetate of lead*, and *mother of pearl*; and I have since discovered that this is the general structure of doubly refracting crystals.

The following is a list of crystals, which I have found to possess more than one axis of polarisation and double refraction.

Arragonite	5 Stilbite
Mica	Cymophane
Topaz	Axinite
Feldspar	Olivine

* The drawings of the rings in *topaz*, which I published in the Phil. Trans. for 1814, are an accurate representation of the phenomena, very near the resultant axes of crystals with more than one axis.

	Epidote	Nitrate of copper
10	Kyanite	Borax of the shops
	Talc	Tartaric acid
	Prehnite	Citric acid
	Dichroite	50 Hyper-oxymuriate of potash
	Lepidolite	Phosphate of soda
15	Mother of pearl	Prussiate of potash
	Indurated talc	Tartrate of potash and soda
	Sulphate of lime	Oxalate of ammonia
	— barytes	55 Super-oxalate of potash
	— strontian	Crystallized Cheltenham salts
20	— lead	Rhomboidal salt
	— magnesia	Murio-sulphate of magnesia and iron
	— iron	Benzoate of ammonia
	— copper	60 Mesotype
	— copper and iron	Oxalic acid
25	— ammonia	Chromic acid
	— potash	Muriate of barytes
	— soda	— mercury
	— zinc	65 — magnesia
	— cobalt	— copper
30	— ammonia and magnesia	Sulphur
	— soda and magnesia	Hydrate of barytes
	— nickel	Super-tartrate of potash
	— manganese	70 Tartrate of potash
	Carbonate of lead	— potash and antimony
35	— soda	Boracic acid
	— ammonia	Succinic acid
	— potash	Chromate of lead
	Nitrate of silver	75 Diallage
	— ammonia	Spodumene
40	— potash	Tincal or native borax
	— lime	Anhydrite
	— bismuth	Sugar
	— mercury	80 Acetate of lead
	— zinc	— copper
45	— strontian	

By examining the preceding list, and comparing it with the list of crystals that have one axis, we may conclude,

1st, that the combinations of sulphuric and tartaric acid with earthy, alkaline, and metallic bases, have two or more axes of double refraction.

2nd, that as only about 23 crystals have one apparent axis of double refraction, while more than 80 have two axes, and as the phenomena of these 23 crystals can be referred to two axes, as will afterwards be shown, the general laws of polarisation and double refraction remain yet to be investigated by experiment.

If we compare the minerals in the preceding list with the most recent table of primitive forms,* it will be found that there is a connection by no means equivocal between the primitive form of a crystal and the number of its axes of extraordinary refraction. In order to establish this curious result, I have drawn up the following Table, in which I have inserted the names of the minerals and the number of their axes opposite to the different primitive forms under which they have been classed.

THREE AXES.	Muriate of soda	Cube.	Two AXES.	Muriate of barytes	Right quadrangular prism with a square base.
	Boracite			Sulphate of magnesia	
	Leucite			Prussiate of potash	
	Analcime			Mesotype	
	Aplome			Sulphate of nickel	
	Phosphate of manganese and iron			zinc	

* The table of primitive forms which I have used, is the one drawn up by Dr. THOMSON in the Article *Crystallography*, in the Edinburgh Encyclopædia, as it contains many of the observations made by HAUX and BOURNON, since the publication of HAUX's *Traité de Mineralogie*.

Two Axes.	Anhydrite Cymophane Prehnite Peridot Stilbite Tartrate of potash	Right quadrangular prism with a rectangular base.	Two AXES.	Sulphate of iron	Rhomboid with an acute summit.
Two Axes.	Sulphate of strontian Sulphate of barytes Mica Talc Spodumene Sulphate of soda Carbonate of potash Tartrate of potash and soda Citric acid	Right quadrangular prism; base a rhomb.	ONE AXIS.	Arseniate of copper Apatite Beryl Emerald Nepheline Sapphire Ruby	Regular hex-aedral prism.
Two Axes.	Sulphate of lime Epidote Axinite	Right quadrangular prism; base an oblique parallelogram.	THREE AXES.	Garnet Blende	Rhomboidal dodecahedron.
Two Axes.	Borax	Oblique quadrangular prism with a rectangular base.	ONE & Two AXES.	Sulphate of potash Fluor spar Muriate of ammonia Alum Spinelle ruby Pleonaste Diamond Ruby copper Nitrate of lead	Bipyramidal dodecahedron.
Two Axes.	Oxalic acid Sugar Sulphur Acetate of copper Tartaric acid Chromate of lead	Oblique quadrangular prism; base a rhomb.	THREE AXES.	Nitrate of potash Sulphate of soda Arragonite* Topaz Carbonate of lead† Sulphate of lead	Regular Octohedron.
Two Axes.	Feldspar Kyanite Sulphate of copper	Oblique quadrangular prism; base an oblique parallelogram.	Two AXES.	Zircon Mellite Idocrase Apophyllite Arseniate of potash	Octohedron in which the pyramids have a rectangular base.
One Axis.	Calcareous spar Bitter spar Quartz Tourmaline	Rhomboid with an obtuse summit.	Two AXES.	Carbonate of soda	Octohedron in which the pyramids have a square base.

* BOURNON makes it a quadrangular rectangular prism with a square base.

† BOURNON makes it a quadrangular prism with a rhomboidal base.

It appears from the slightest examination of the preceding Table, that all the crystals with *one*, *two*, or *three* axes, range themselves under their particular primitive forms, and that the only exceptions to this general fact, are *Idocrase* and *Apophyllite*, which we have no doubt will be found to have primitive forms, different from those assigned to them by HAUY. All the crystals with one axis have for their primitive forms a hexaedral prism, a rhomboid with an obtuse summit, and an octohedron in which the pyramids have a square base. All the crystals with three axes, have the cube, the regular octohedron, and the rhomboidal dodecahedron for their primitive forms; and all the crystals with two axes, crystallize in the various other forms given in the Table.

Hence we are furnished with the means of deducing the number of axes in crystals from their primitive form, and of approximating to the primitive form when the number of axes is given, by excluding certain other primitive forms which belong to a different number of axes. If HAUY and BOURNON have given these forms correctly for the following substances, in which I have not detected the number of the axes, they will be arranged as in the annexed Table.

ONE AXIS.	TWO AXES.
Tungstate of lime.	Euclase.
Octohedrite.	Calamine.
Ruby silver.	Corundum.
Cinnabar.	Staurolite.
5 Carbonate of strontian.	5 Datolite.
Dioprase.	Sphene.
Harmotome.	Titanite.
Chabasie.	Wernerite.
Cryolite.	Meionite.
10 Phosphate of lead.	10 Paranthine.
Glacial sulphuric acid.	Acetate of barytes.
Carbonate of barytes.	Calomel.
Pinite.	Arseniate of potash.
Spinellane.	Green carbonate of copper.

1. *On the form of the rings or isochromatic curves, and on the nature of the tints in crystals with more than one axis.*

We have already seen that when a crystal has only one apparent axis of double refraction, the isochromatic curves, or lines of equal tint are perfect circles having the axis of extraordinary refraction passing through their centres; but when these curves are the result of two separate axes they assume the more complicated form represented in Pl. XV. fig. 4.

If we transmit polarised light in every possible direction through a crystal, COD \circ , Pl. XV. fig. 5, (which is a section of the sphere in Fig. 4, through the great circle COD \circ .) we shall find that there are two diameters P p , P' p' in which there is neither polarisation nor double refraction. To these lines I have given the name of *resultant axes or diameters of no polarisation*, P,P', p , p' Pl. XV. figs. 4 and 5, being the poles of no polarisation. Each of these poles is surrounded with similar sets of rings, the tints of which commence at P, P' p , p' increase towards O, C, D, A, and B, and reach their maximum at A and B. When the distance PP' or the inclination of the diameters of no polarisation is considerable, the rings near P,P are almost circular, but the circularity soon ceases as they recede from the pole. If PP' is less than 90°, the tints at C,D, which are always equal, are higher than those at O, and when P,P is exactly 90°, the tints at C,O,o, D are equal, and the rings are symmetrical round P,P'.

As PP' is always less than 90°, the great circle ACBD may be called the *Equator of maximum double refraction or polarisation*, since the double refraction and the polarisation are always greatest in this line. The great circle AOB \circ ,

Pl. XV. figs. 4 and 5, may be called the *meridian of direct polarisation*, because the tints ascend directly from O, σ to A and B, and the great circle COD σ the *meridian of inverse polarisation*, not only because the tints descend from O to P and P', and ascend from P to C and from P to D, but because there is a real inversion in the character of the tints on each side of PP'.*

In *mica*, *topaz*, and other crystals, where the distance PP' is 45° and upwards, the system of rings represented in Pl. XV. fig. 4, cannot easily be seen at once; but in *nitre* and other crystals where PP' is very small, the whole system is finely developed, and every individual curve may be examined with attention.

In order to observe these rings to advantage, let a plate of nitre ACBD, Pl. XV. fig. 6, about $\frac{1}{2}$ or $\frac{1}{5}$ of an inch thick, be cut perpendicular to the axis of the hexaedral prism. If this plate is exposed to polarised light, so that either AB or CD is in the plane of primitive polarisation; and if the transmitted light is analysed with an achromatic prism of calcareous spar, the extraordinary image will exhibit the system of rings shown in Pl. XVI. fig. 7, while the ordinary one will exhibit a system exactly complementary.

The poles P, P' of no polarisation, are distant $5^\circ 20'$, whatever be the thickness of the plate; and through them passes one of the branches CD of the rectangular cross A, B, C, D. The breadth of each ring is least between C and P and D

* The preceding names given to the three great circles, have been drawn from the most prominent physical characters which belong to them. Other names, such as the *meridian of the principal axis*, *the meridian of the secondary axis*, &c. would have been preferable, had they not involved an hypothesis; for it will afterwards be seen that the phenomena may be equally well explained by axes varying both in number and position.

and P, and gradually increases towards AB, owing partly to the different thicknesses at which the light is transmitted through the plate.

When the plate of nitre is turned round its axis, the black cross ABCD immediately breaks up, according to laws which will afterwards be explained ; and when AB forms an angle of 45° with the plane of primitive polarisation, the system of rings has the appearance shown in Pl. XVI. fig. 8. The rings themselves have suffered no change by this change of position in the plate ; but the black cross is separated into two hyperbolical branches M'P'N', MPN. When the thickness of the plate is above $\frac{3}{10}$ ths of an inch, the portions of the rings included between the two hyperbolic branches, are confounded into one mass of white light, while those between C and P, or D and P, are distinctly visible. The concave sides of the hyperbolic branches are strongly fringed with a red and yellow colour, while the convex or inner sides are equally affected with blue rays. By diminishing the thickness of the plate, the rays gradually appear between the hyperbolic branches, but they consist only of pink and green tints, like those in the 5th order of NEWTON's scale ; and even when the rings almost cease to appear by a great diminution of thicknesses, the tints, though at the very commencement of the first order, are not the same within as without the poles of no polarisation. In the position of Pl. XVI. fig. 7, however, and when the tint at O, in the position of Pl. XVI. fig. 8, is only blue of the first order, this irregularity of the tints is almost imperceptible, and the system of rings is distinguished from that produced by crystals with one axis only, by a slight degree of ellipticity. This approximation to the

system shown in Pl. XV. fig. 1, arises from the great attenuation of the crystallized plate, which is no longer capable of developing the tints between the poles of no polarisation; and it is only from the elliptical form of the curves that we can in this state recognise the existence of two axes. Even this ellipticity will disappear by a farther attenuation of the plate; and the crystal deprived as it were of one of its axes, appears to act upon light, exactly like beryl or calcareous spar.

The phenomena which have now been explained, I have found, under various modifications which will afterwards be described, in all the crystals that have more than one axis. The irregularities in the tints are, in every case, developed by increasing the thickness of the plate, and affect the rings which are formed at a considerable distance from the poles of no polarisation. They are analogous to the tints seen along the axis of rock-crystal; and as the secondary forces by which they are produced, conceal the regular action of the principal forces, we must abstract these secondary effects in determining the law according to which the legitimate tints are developed.

2d. On the character, the number, and the position of the axes, by which the tints are produced.

The most important physical circumstance which distinguishes the system of rings formed by different crystals, is the magnitude of the arch PP' , or the inclination of the resultant axes, or diameters of no polarisation; for it is from this angle alone that we can deduce the relative intensity of the real axes, if we suppose them rectangular; or their mutual inclination, if we suppose them equal and inclined.

It would occupy too much room to detail the various practical methods to which I have been obliged to resort in determining the position and inclination of the diameters of no polarisation. In some crystals this process is sufficiently simple, as in topaz, mica, nitre, talc, mother of pearl, prussiate of potash &c. but in other crystals it is extremely difficult, particularly when the angle of inclination is great, and when the specimen is small and its crystalline form indistinct.

The following table contains this angle for 49 crystals. The measures were taken with great care, but some of them are only estimated, and others will admit of correction by the use of better specimens than those I was able to procure.

TABLE of the inclination of the resultant axes of forty nine crystals.

Sulphate of nickel, certain specimens	25	Sugar	50	0
Carbonate of lead	3	Sulphate of strontites	50	2
Nitrate of potash	5	Muri-o-sulph. magn. and iron	51	16
Talc	5 20	Sulph. ammon. and magnes.	51	22
5 Mother of pearl	7 24	Phosphate of soda	55	20
Hydrate of barytes	11 28	30 Sulphate of lime	60	0
Mica, certain specimens, about	13 18	Oxynitrate of silver	62	16
Arragonite	14 0	Feldspar	63	0
Prussiate of potash	18 18	Topaz	65	0
10 Cymophane	19 34	Sulphate of potash	67	0
Borax	27 51	35 Carbonate of soda	70	1
Anhydrite	28 42	Acetate of lead	70	25
Sulphate of magnesia	28 7	Citric acid	70	29
15 Tincal	37 24	Tartrate of potash	71	20
Nitrate of zinc, estimated at	37 42	Tartaric acid	79	0
about	38 48	40 Tartrate potash and soda	80	0
Stilbite	40 0	Carbonate of potash	80	30
Sulphate of nickel	41 42	Kyanite	81	48
Carbonate of ammonia	42 4	Hyper-oxymur. potash	82	0
20 Mica	43 24	Muriate of copper	84	30
Lepidolite	45 0	45 Epidote, about	87	0
Benzoate of ammonia	45 8	Peridot	87	56
Sulphate of zinc	44 28	Crystallized Chelt.-salts	88	14
ammonia	49 42	Succinic acid, estimated at	90	0
		Sulphate of iron	90	0

In examining the preceding table, we cannot fail to be struck with the uniform distribution of the angles over the quadrant; a circumstance which contributes to render this new property of crystals a most distinct and valuable physical character in mineralogy.

Having thus determined the angles of the resultant axes, we must now proceed to ascertain the position, the number, and the relative intensities of the real axes, by the combination of whose action the various tints are produced; and in doing this we shall take *mica* as an example.

M. BIOT has announced it as a demonstrated physical truth, that mica has two repulsive axes, one in the plane of the laminæ, and the other perpendicular to the laminæ; and that the polarising force of the first is to that of the second as 100 to 677.* As the experiments from which he deduced this system of forces, consist of observations on the tints, merely in the two rectangular directions, AOB, COD, Pl. XV. fig. 4, we cannot admit them as any thing approaching to evidence for the conclusion which he has drawn; for it is possible that the tints developed in the four quadrants, ACO, ADO, CBO, DBO, may be quite incompatible with such an arrangement. I have observed the tints produced by mica in almost every part of the four quadrants; and it follows from the general law of polarisation which I have discovered, that there is no particular system of forces indicated by the phenomena; but

* “Tous ces résultats étant exactement conformes à ce qu'on observe, il est bien vraisemblable que le système des forces polarisantes du mica est ainsi combiné; mais pour *changer cette présomption en certitude*, il faut exprimer par le calcul les effets d'un pareil système, et voir si la marche des teintes, observées sous chaque incidence, s'y conforme exactement.” BIOT *Traité de physique*, tom. iv. p. 557.

that various systems may be proposed which are equally consistent with the observed tints. The number and position of the axes of mica, the rectangularity of the axes, and the nature of the forces which emanate from them, as given by M. BIOT, are therefore entirely hypothetical results.

The phenomena of mica may be explained by the following hypothetical arrangements of the axes.

1. *Rectangular axes.*

1st, By *two unequal negative axes*, Oo, AB, Pl. XV. fig. 4, whose relative intensities are as 100 to 677.

2d, By *a negative axis* Oo, and *a positive axis* CD, whose intensities are as 100 to 173.

3d, By *two unequal positive axes*, CD, AB, whose intensities are as 100 to 117.

2. *Oblique axes.*

4th, By *two equal positive axes*, RS, TV, inclined to each other at an angle of $85^{\circ} 26'$ and

5th, By *two equal negative axes* X, Z, inclined to each other at an angle of $42^{\circ} 2'$.

Hitherto we have supposed that the axes from which the forces emanate, are only two in number; but it will appear from a subsequent part of this paper, that any one axis may be resolved into any even or odd number of axes, by which the phenomena may be explained. All that we know, in short, is that a certain polarising force is exerted at a particular point, but we have no means of ascertaining either the number or the direction of the forces of which it is the resultant. Certain physical circumstances, however, which

will afterwards be noticed, (relative principally to the tints, and the coincidence between some of the hypothetical axes and the principal lines in the crystal,) may lead us to prefer these axes to others; but however plausible the grounds of our preference may be, we must take care not to admit these hypothetical deductions among the number of demonstrated physical truths.

We have already stated that M. BIOT has divided crystals into two classes, attractive and repulsive; and that this division is quite hypothetical, even when applied to crystals with one axis. To the case of crystals with two axes, the classification is wholly inapplicable, for it has been shown, that there are no means of ascertaining either the nature, the number, or the position of the axes. Still, however, the terms *positive* and *negative* may, in most cases, be conveniently employed to mark the principal resulting force which the crystal exhibits. Thus, if we suppose O, Pl. xv. fig. 4, which is the middle point between the two nearest poles of no polarisation to be the position of the principal axis, then mica will be negative, because the tints from O to A, from O to B, from P to C, and from P' to D, have all a negative character; and topaz will be positive, because the same tints have a positive character. In the case of crystals, however, where PP is 90° , even this limited application of the terms negative and positive entirely fails, and these crystals cannot be considered as belonging to one class more than to another.

3d. On the general law of the tints for all crystals with one or more axes.

When a crystal has one axis of double refraction, the tints are disposed on the surface of the sphere, in regular concentric circles around the real or resultant axes of the crystal; and the intensity of the tint at any point of the spherical surface, is equal to $\text{Sin.}^* \phi$, ϕ being the distance of the point from the pole of the axis, and the maximum tint in the equator of double refraction being considered as unity. This value of the tint was deduced by M. BIOT, from experiments on sulphate of lime, rock crystal, Iceland spar, and some specimens of mica, before he was acquainted with the discovery of the system of concentric rings; but it was obviously not entitled to any confidence as a general principle, not only from its having been deduced from such a small number of crystals, but from its not representing the phenomena in the very crystals from which it was deduced. In sulphate of lime, for example, $\text{Sin.}^* \phi$ is every where erroneous as the value of the tints. This error indeed is very small in particular azimuths when the distance of the tint from the pole, of what M. BIOT calls the axis, exceeds 30° , but in other azimuths, such as that of 90° , the error is enormous, and in the great circle passing through the plane of the laminæ, the phenomena have no connection whatever, with this law. M. BIOT himself perceived the utter incompetency of the expression $\text{Sin.}^* \phi$ to represent the phenomena in the vertical azimuths, and has expressed the aberrations which he observed in these directions by complicated empirical formulæ. These formulæ, however, though they represent M. BIOT's,

do not at all represent the actual phenomena, for M. BIOT never made a single observation on the tints in the direction of the laminæ, the only direction in which they could be investigated below 30° , from the supposed pole. Had the experiments been made in this way, he would have found that all the observations, instead of being owing, as he supposes, to secondary forces arising from the unequal superposition of the laminæ, are the legitimate results of two axes of double refraction. The experiments of this philosopher upon sulphate of lime must therefore be set aside as incompetent to determine the accuracy of $\text{Sin.}^2\phi$, as the expression of the tints in crystals with one axis.

As rock crystal possesses secondary forces which interfere with the action of the principal axis, and as there is reason for believing that its apparent axis is only the resultant of two equal negative axes, the value $\text{Sin.}^2\phi$ cannot be deduced from the valuation of the tints which it develops.

The experiments made by M. BIOT upon calcareous spar, previous to his knowledge of the coloured rings, were better fitted to afford an expression of the variation of the tints; but when we consider that his mode of observation was such that, when applied to arragonite, it gave $\text{Sin.}^2\phi$ as the law of the tints, and that this mineral has two axes, and cannot therefore have its tints regulated by such a law, we are forced to conclude, that this mode of observation is insufficient even when applied to calcareous spar.

Some new method, therefore, of studying the phenomena of double refraction and polarisation was wanting, in order to determine with certainty, whether any crystal had one or more axes; and what is the law according to which the tints vary in crystals with one axis. By the old mode of observation,

MALUS concluded that *arragonite* and *sulphate of barytes* were crystals with one axis, whereas they have both two axes, and, by M. BIOT's mode of observation, he concluded that *sulphate of lime*, *arragonite*, *topaz*, *sulphate of barytes*, *sulphate of strontian*, and *feldspar*, had only one axis, whereas they have all two distinct axes of double refraction.

The method of observation which I have always employed, consists in observing the system of coloured rings which are seen along the axes of crystals with one axis, and along the resultant axes of crystals with two axes. These rings I first discovered in mica and topaz, about the end of the year 1812, and early in 1813 I discovered in beryl and several other minerals with one axis, the system of rings peculiar to crystals of that class. Dr. WOLLASTON discovered the same system of rings in calcareous spar in 1814; and long after this discovery was made, M. BIOT examined the phenomenon, and showed by measuring the diameters of the rings that in this crystal the variation of the tints was expressed by $\text{Sin.}^* \phi$. By similar measurements of the rings in *zircon*, *ice*, *beryl*, *emerald*, *sapphire*, *ruby*, *rubellite*, *tourmaline*, *apatite*, *vesuvian*, *mellite*, *nepheline*, *muriate of lime*, *muriate of strontian*, *arseniate of copper*, I have found that the expression $\text{Sin.}^* \phi$ applies within the limits of the error of experiments to all these crystals, which, excepting *hydrate of magnesia* and *quartz*, are the only crystals known to have but one axis of double refraction. I therefore consider myself as entitled to set out with this formula, as an expression of the tints for all crystals with one axis, whether their action is of a positive or a negative character.

In proceeding to explain the general law which I have discovered for determining the tints in crystals with any

number of axes, let us suppose that ABC, Pl. xvi. fig. 9, represents the quadrant of a spherical surface, such as we have described in p. 266; and that the position of G, one of the resultant axes, where the tint is nothing, has been carefully determined by experiment, it is required to find the tint at any point E by the action of certain polarising forces which are in *equilibrio* at the point G. If we now suppose,

1st. That the tints are produced by forces emanating from *two negative axes* whose poles are C, A, it is obvious that their relative intensities must be in the ratio of $1 : \frac{1}{\sin.^2 GC}$, GC representing half the inclination of the diameters of no polarisation. For as the tint at G produced by A is equal to the tint produced at the same point by C, and since the tint produced there by the axis A is its maximum tint, AG being 90° , then the maximum tint produced by C will be found by the analogy $\sin.^\circ GC : \text{Rad.}^2 = 1 : \frac{1}{\sin.^2 GC}$.

2d. If we suppose that the forces emanate from *two positive axes* A, B, A being greater than B, then the relative intensities must be as $1 : \frac{1}{\cos.^2 GC}$.

3d. If the forces emanate from two axes B, C, one of which is positive and the other negative, the intensity of B must be to that of C as $\sin.^\circ GC : \cos.^\circ GC$.

Through E draw three great circles AEF, BE and CE, Pl. xvi. fig. 9. and let

T = tint required at the point E.

θ = the arch between the point E and the axis C.

ϕ = the arch between the points E and B.

a = the tint produced separately at E by the greater axis.

b = the tint produced separately at E by the lesser axis.

ψ = the angle of the forces.

π = the angle B,EF.

ω = the angle CEF.

A = the arch FC, or the angle CAF, or the *azimuth* on the great circle BGC passing through the poles of no polarisation.

D = the arch FE, or the declination or distance of the point E from the same great circle.

ζ = half the difference of the angles at the base or at the diagonal of the parallelogram of forces. Then

1. When the two axes are B,C in the plane passing through the diameters of no polarisation, we have

$$\cos. \theta = \cos. A \times \cos. D.$$

$$\cos. \phi = \sin. A \times \cos. D.$$

2. When the two axes are C,A in a plane perpendicular to the plane passing through the diameters of no polarisation,

$$\cos. \theta = \cos. A \times \cos. D.$$

$$\phi = 90^\circ - D.$$

Then we have, in general, whether the axes are A,C or B,C

$$\cos. \omega = \frac{\text{Tang. } D}{\text{Tang. } \theta}.$$

$$\cos. \pi = \frac{\text{Tang. } D}{\text{Tang. } \phi}.$$

When B,C are the two axes, either both positive or both negative, $\psi = 2 \frac{\pi}{\pi + \omega}$.

When A,C are the axes either both positive or both negative, $\psi = 2(180^\circ - \omega) = 2\omega$.

When B,A are the axes either both positive or both negative, $\psi = 2(180^\circ - \pi) = 2\pi$

When B,C are the axes, the one positive and the other negative,

$$\psi = 180^\circ - 2\pi + \omega = 2\pi + \omega$$

When A and C are both positive, or both negative, and $C > A$

$$a = \text{Sin.}^2 CE, \text{ and } b = \text{Sin.}^2 EA \times \text{Sin.}^2 GC$$

When A and B are both positive, or both negative, and $A > B$

$$a = \text{Sin.}^2 AE, \text{ and } b = \text{Sin.}^2 BE \times \text{Cos.}^2 GC$$

When B and C are the one positive, and the other negative,

$$a = \text{Sin.}^2 EC \times \text{Cos.}^2 GC, \text{ and}$$

$$b = \text{Sin.}^2 BE \times \text{Sin.}^2 GC.$$

The tints produced separately by each axis being thus determined, the tint resulting from their joint action will be found to be the diagonal of a parallelogram, whose sides are a, b and whose angle is ψ . In order to find this diagonal,

$$\text{We have } \text{Tang. } \zeta = \frac{a - b \text{ Tang. } \frac{1}{2}\psi}{a + b}, \text{ and}$$

$$\zeta + \frac{1}{2}\psi = \text{Greater angle at the base};$$

$$\text{Hence } T = \frac{a \text{ Sin. } \psi}{\text{Sin. } (\zeta + \frac{1}{2}\psi)}.$$

$$\text{When } a = b \text{ then } T = 2a (\text{Cos. } \frac{\pi}{\pi + \omega})$$

When $a = b$, and the axes equal, then $\pi = \omega$ and

$$T = 2a (\text{Cos. } 2\pi) \text{ or } T = 2a (\text{Cos. } 2\omega) \text{ and since } \phi = \theta$$

$$T = 2 \text{ Sin.}^2 \phi (\text{Cos. } 2\pi)$$

$$\text{When } \psi = 90^\circ \text{ then } T = \sqrt{a^2 + b^2}$$

$$\text{When } \psi = 180^\circ \text{ } T = a - b$$

$$\text{When } \psi = 270^\circ \text{ } T = a + b$$

This general law of the tints may be expressed in the following manner: *the tint produced at any point of the sphere by the joint action of two axes, is equal to the diagonal of a parallelogram, whose sides represent the tints produced by each*

axis separately, and whose angle is double of the angle formed by the two planes passing through that point of the sphere and the respective axes.

If the crystal has three or more axes, the resulting tint produced from any two of them may, in like manner, be combined with the third, and this resulting tint with the fourth, till the general resultant of all the forces is obtained.

If the number of axes with given intensities exceeds *two*, they may be combined by the methods explained in the next section, till they are reduced to two axes, with new relative intensities ; and the resultant of all the axes will be obtained by the calculation of the diagonal of a single parallelogram.

The law which we have now explained is obviously deduced from no empirical data, but is rigorously physical, and is founded upon the same principles which regulate the combination of all other mechanical forces. The accuracy with which it represents the complicated system of tints is very wonderful, and cannot fail to recommend it to the reception of philosophers as a true law of nature. In establishing its conformity with the actual phenomena, I shall not content myself with examining it by means of my own experiments. I shall submit it to the severe ordeal of M. BIOT's measurements of the tints of *sulphate of lime*, taken long before the discovery of the law, and which he considered as the result of irregular action depending upon imperfect crystallization. To those who may desire farther evidence, a still more decisive trial may be offered ; a trial too, in which the eye itself is capable of recognising the perfect identity between the observed and calculated results. If we compute all the tints,

by means of the law, for any crystal in which the rings round the resultant axes can be seen at one view, and project them upon paper, after they are reduced to different thicknesses, corresponding to the oblique transit of the rays through the parallel plate, we shall have a representation of the rings actually observed, expressing in the most accurate manner all the inversions of the tints, and exhibiting the points of contrary flexure, and the innumerable varieties of form which the curves assume.

In order to determine the tints of sulphate of lime at great obliquities, M. BIOT placed the laminæ in a tube shut up at both ends by plates of glass ; and by means of a metallic rod he was able to fix them at any angle with the axis of the tube. He then filled the tube with water, or oil of turpentine, and observed the tints at inclinations as high as seventy-eight degrees, two minutes. In this way he constructed a formula which represented these observations, and by means of this formula he computed the following table, in which I have reduced his numbers, in order to represent the tints at equal thicknesses, by dividing them by the secants of the angles of refraction.

TABLE of the tints of sulphate of lime in different azimuths reduced from M. BIOT's experiments.

Angle of refraction	Angular dist. from the axis.	Azimuth of 0°	Azimuth of 22° 30'	Azimuth of 45°	Azimuth of 67° 30'	Azimuth of 90°
0°	90°	1.0000	1.0000	1.0000	1.0000	1.0000
10	80	0.9768	0.9787	0.9838	0.9905	0.9944
20	70	0.9105	0.9164	0.9352	0.9615	0.9756
30	60	0.8101	0.8213	0.860c	0.9117	0.9434
40	50	0.6885	0.7047	0.7664	0.8523	0.9002
50	40	0.5559	0.5751	0.6607	0.7794	0.8482
60	30	0.4241	0.4409	0.5483	0.6982	0.7938
70	20	0.3027	0.3043	0.4121	0.5949	0.7337
80	10	0.2077	0.1599	0.2501	0.4060	0.6184
90	0					

In determining the tints by new experiments, I adopted two different methods of investigation. By one of these, which was analogous to that of M. BIOT, I observed the tints for all angles of refraction up to 60° , and by another mode I observed the remaining tints up to 90° , and I endeavoured to avoid as much as possible the secondary effects produced by obliquity.

The first of these methods is shown in Pl. xvi. fig. 11, where ABCD is a plate of sulphate of lime, whose natural surfaces, AB, CD, are parallel to the plane passing through the resultant axes. Upon these surfaces I cemented two prisms of crown glass M,N, by a thin and equal film of Canada balsam, and having placed them upon the goniometer, I was able to observe the tints with great correctness at angles of refraction considerably beyond those which could be obtained in air. At angles of refraction between 60° and 90° , I resorted to the second method, which consisted in transmitting the polarised light through the parallel faces AD, AC. This method is attended with peculiar difficulties; and much perplexing labour must be submitted to before good plates can be obtained, as the laminæ are constantly separating from each other; and by admitting the materials employed in grinding and smoothing the surfaces, the transparency of the plate is destroyed. I found it necessary, indeed, to bind the laminæ together by wax, sometimes by plates of wood, and at other times by placing them in a small hand vice. By these means my experiments at last succeeded, and all the mysterious actions of sulphate of lime, with the origin and classification of which M. BIOT had been so much perplexed, were immediately unravelled. The two resultant axes and their attendant rings were observed at an angular distance of 60° ; the relative intensities of the two axes, from which all the irregularities arose, were thus given,

and the actual progress of the tints became a matter of simple observation. In this way I constructed the following table.

Angle of refraction	Angular dist. from the axis.	Azimuth of 0°	Azimuth of $22^\circ 30'$	Azimuth of the resultant axis 30°	Azimuth of 45°	Azimuth of $67^\circ 30'$	Azimuth of 90°
0°	90°	1.0000	1.0000	1.000	1.0000	1.0000	1.0000
10	80	0.978	0.979	0.980	0.984	0.992	0.995
20	70	0.913	0.919	0.927	0.938	0.962	0.973
30	60	0.811	0.824	0.835	0.861	0.913	0.939
40	50	0.689	0.710	0.729	0.768	0.853	0.898
50	40	0.558	0.576	0.588	0.662	0.782	0.850
60	30	0.435	0.438	0.448	0.538	0.719	0.810
70	20	0.340	0.294	0.304	0.410	0.659	0.775
80	10	0.274	0.125	0.154	0.305	0.614	0.760
90	0	0.250	0.105	0.000	0.246	0.597	0.753

In the preceding table I have not given the value of the tints to more than three decimal places, as even the third decimal place is partly the result of estimation.

To those who may repeat these experiments, it will be necessary to state, that in almost all crystals with two axes, the tints in the neighbourhood of the resultant axes, when the plate has a considerable thickness, lose their resemblance to those of NEWTON's scale, as will be more minutely described in another paper. The rings, however, are perfectly formed; and the numbers in the table are the values of the tints deduced from their position, and not from their actual colour. Thus, in the third ring or order of colours, reckoned from the resultant axes, I call the value of the middle point about 17, although the tint is not a yellowish green, as in NEWTON's scale. This mode of proceeding is strictly correct, for the cause which prevents the tint from being a yellowish green, disappears in general by diminishing the thickness of the plate.

In comparing these observations with the general law, we shall suppose that one of the axes has the same situation in

the laminæ as the line which M. BIOT calls the axis,* when the other axis is perpendicular to the plane of the laminæ. These positions are indicated in Pl. xv. fig. 4, by O and A, and in fig. 9, by C and A. Since GC, therefore, or half the distance of the resultant axis is 30° , we shall have the relative intensity of the axis C, to that of A, as $\frac{1}{\sin^2 GC}$: or as 4 to 1. With these data the following table has been calculated.

Angle of refraction.	Angular distance from the axis.	Azimuth of 0° .	Azimuth of $22^\circ 30'$.	Azimuth of the resultant axis, 30° .	Azimuth of 45° .	Azimuth of $67^\circ 30'$.	Azimuth of 90° .
0°	90°	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
10	80	0,9775	0,9796	0,9811	0,9848	0,9903	0,9925
20	70	0,9124	0,9202	0,9258	0,9401	0,9614	0,9707
30	60	0,8125	0,8272	0,8385	0,8660	0,9160	0,9375
40	50	0,6901	0,7093	0,7254	0,7729	0,8573	0,8967
50	40	0,5600	0,5755	0,5936	0,6593	0,7901	0,8533
60	30	0,4375	0,4356	0,4507	0,5339	0,7221	0,8125
70	20	0,3378	0,2980	0,3018	0,4071	0,6615	0,7792
80	10	0,2726	0,1238	0,1511	0,2983	0,6190	0,7575
90	0	0,2500	0,1035	0,0000	0,2500	0,6036	0,7500

The agreement between the numbers in this and the preceding table, will convey an idea of the accuracy with which the law accommodates itself to all the capricious changes of the tints. I have compared it also with numerous experiments made on *nitre*, *mica*, *topaz*, *sulphate of iron*, &c. embracing all the varieties in the inclination of the resultant axes, and have found it equally accurate in all these crystals. The projection of the calculated results, indeed, compared with the system of rings where their curvature is the most variable, may be considered as an ocular demonstration of the correctness of the law.

* It is a curious fact, that sulphate of lime is the only crystal, out of a great number, in which the principal axis does not coincide with any of the prominent lines in its primitive form, as ascertained by HAUX. Does not this prove incontestibly that its primitive form is at present undetermined?

SECT. IV. *On the resolution and combination of polarising forces, and the reduction of all crystals to crystals with two or more axes.*

It has been remarked by M. BIOT in one of his latest memoirs, that it has been established by LAPLACE, that if the Huygenian law is admitted, the extraordinary refraction is *necessarily* produced by a repulsive force; and the same philosopher has also asserted, that from the opposite character of the tints produced by beryl and quartz, there results *necessarily* this alternative: “Either the forces which produce the extraordinary refraction are repulsive in one of the classes and attractive in the other, or they are repulsive in both; but they turn the axes of the luminous molecules in directions inversely rectangular. New experiments have proved to me that the first mode is that which nature realises.” *

This view of the subject of double refraction and polarisation has always appeared to me erroneous. I have never been able to perceive that the phenomena of calcareous spar were *necessarily* referable to a repulsive force, or that nature had restricted herself to any of the alternatives which have just been stated. These opinions have acquired new strength as I advanced in the inquiry, and I trust I shall be able to demonstrate, not only that the phenomena of double refraction and polarisation may be explained by forces or combinations of forces different from those which have been given by LAPLACE and BIOT, but that there are certain analogies of nature, and certain physical circumstances in the phenomena, which

* *Mem. de l' Institut. Lu, 2 Jan. 1815.*

may lead us to select one combination of forces in preference to others, as the means which nature has employed in the accomplishment of her purposes.

If we consider a material particle in motion as under the influence of forces, the nature and the source of which are unknown, we may ascribe any change of direction which it experiences, either to a single attractive, or a single repulsive force, emanating from different sources ; or we may regard it as the resultant of a variety of forces of the same, or of opposite characters. In the phenomena of the solar system, a repulsive force is necessarily excluded by the simplest considerations. In the reflection of light, the return of the ray cannot, without the most manifest absurdity, be ascribed to an attractive force residing without the reflecting surface ; and for the same reason, the refraction of the transmitted light cannot be considered as the effect of a repulsive force existing without the transparent medium. But the case is quite different in the phenomena of double refraction and polarisation. There are here no prominent physical circumstances which can lead us to a general determination of the nature of the forces. The deviation of the extraordinary ray in beryl, may be the result of a repulsive force emanating from the axis of the prism, or of an attractive force emanating from two equal rectangular axes lying in a plane perpendicular to the axis of the prism, or of various other combinations of forces, either of the same or of opposite names.

We shall now proceed to demonstrate the first of these positions, namely, that the action of two equal rectangular axes of a positive character, as calculated by the law of polarisation already explained, is the same as the action of

one negative axis of the same intensity as either of the other two, and placed at right angles to the plane of the positive axes.

Let A,B,C, fig. 9, be the poles of the three axes of which B,C are the positive axis, and A the negative axis.

The tint produced at any point E by the axis A alone, is of the same intensity, and the same character, as the tint that would be produced at the same point by the positive axes B,C acting jointly. Through E draw the great circles AEF, BE, and CE, and call $EF=x$, $BE=\phi$, $EC=\theta$, $BF=m$, $CF=n$, $BEF=\pi$, $CEF=\omega$. Then as the tint produced at E by the axes A alone is $\sin^2 AE$ or $\cos^2 x$, we must show that the resulting tint produced by the two axes B,C, is also equal to $\cos^2 x$. By spherical trigonometry we have

$$\sin^2 \phi \times \sin^2 \pi = \sin^2 m$$

$$\sin^2 \theta \times \sin^2 \omega = \sin^2 n$$

But since $m + n = 90^\circ$ $\sin^2 n = \cos^2 m$. Hence

$$\sin^2 m + \sin^2 n = 1$$

and by adding together the two first equations we have

$$\sin^2 \phi \times \sin^2 \pi + \sin^2 \theta \times \sin^2 \omega = 1.$$

Again, since $\text{Tang. } \phi = \frac{\sin. \phi}{\cos. \phi}$, and $\text{Tang. } x = \text{Tang. } \phi \times \cos. \pi$

and $\cos. \phi = \cos. x \times \cos. m$, we obtain by substitution

$$\text{Tang. } x = \frac{\sin. \phi \times \cos. \pi}{\cos. x \times \cos. m} \text{ and}$$

$\text{Tang. } x \times \cos. x \times \cos. m = \sin. \phi \times \cos. \pi$; but since

$\text{Tang. } x \times \cos. x = \sin. x$ we have

$\sin. x \times \cos. m = \sin. \phi \times \cos. \pi$, and by the same reasoning

$$\sin. x \times \cos. n = \sin. \theta \times \cos. \omega.$$

Hence, after squaring both equations and adding them together, we have

$$\overline{\text{Sin.}^2 x \times \text{Cos.}^2 m} + \overline{\text{Sin.}^2 x \times \text{Cos.}^2 n} = \overline{\text{Sin.}^2 \phi \times \text{Cos.}^2 \pi} + \overline{\text{Sin.}^2 \theta \times \text{Cos.}^2 \omega}$$

$$\text{and } \overline{\text{Sin.}^2 x} = \frac{\overline{\text{Sin.}^2 \phi \times \text{Cos.}^2 \pi} + \overline{\text{Sin.}^2 \theta \times \text{Cos.}^2 \omega}}{\overline{\text{Cos.}^2 m \times \text{Cos.}^2 n}}$$

$$\text{But } \overline{\text{Cos.}^2 m \times \text{Cos.}^2 n} = 1. \text{ Hence}$$

$$\overline{\text{Sin.}^2 x} = \overline{\text{Sin.}^2 \phi \times \text{Cos.}^2 \pi} + \overline{\text{Sin.}^2 \theta \times \text{Cos.}^2 \omega}.$$

$$\text{But since } \overline{\text{Cos.}^2 \pi} = \overline{\text{Cos.} 2 \pi} + \overline{\text{Sin.}^2 \pi} \text{ and}$$

$$\overline{\text{Cos.}^2 \omega} = \overline{\text{Cos.} 2 \omega} + \overline{\text{Sin.}^2 \omega} \text{ we have}$$

$$\overline{\text{Sin.}^2 x} = \overline{\text{Sin.}^2 \phi \times \text{Cos.} 2 \pi} + \overline{\text{Sin.}^2 \phi \times \text{Sin.}^2 \pi} + \overline{\text{Sin.}^2 \theta \times \text{Cos.} 2 \omega} + \overline{\text{Sin.}^2 \theta \times \text{Sin.}^2 \omega}$$

$$\text{But } \overline{\text{Sin.}^2 \phi \times \text{Sin.}^2 \pi} + \overline{\text{Sin.}^2 \theta \times \text{Sin.}^2 \omega} = 1; \text{ hence}$$

$$\overline{\text{Sin.}^2 x} = \overline{\text{Sin.}^2 \phi \times \text{Cos.} 2 \pi} + \overline{\text{Sin.}^2 \theta \times \text{Cos.} 2 \omega} + 1, \text{ and}$$

$$\text{Since } \overline{\text{Cos.}^2 x} = 1 - \overline{\text{Sin.}^2 x} \text{ we have}$$

$$-\overline{\text{Cos.}^2 x} = \overline{\text{Sin.}^2 \phi \times \text{Cos.} 2 \pi} + \overline{\text{Sin.}^2 \theta \times \text{Cos.} 2 \omega}$$

Now in the parallelogram of forces, the two forces AC,AD (Pl. xvi. fig. 12) are $\overline{\text{Sin.}^2 \phi}$, $\overline{\text{Sin.}^2 \theta}$ when the axes are of equal intensity, that is $\overline{\text{AC}} = \overline{\text{Sin.}^2 \phi}$, and $\overline{\text{AD}} = \overline{\text{Sin.}^2 \theta}$. But

$$\overline{\text{Sin.}^2 \phi} : \overline{\text{Sin.}^2 \theta} = \overline{\text{Sin.}^2 \omega} : \overline{\text{Sin.}^2 \pi}, \text{ and the}$$

$$\text{Angle CAD} = 2\pi + 2\omega. \text{ Hence}$$

$$\overline{\text{CAB}} = 2\pi \text{ and}$$

$$\overline{\text{BAD}} = 2\omega$$

$$\text{Consequently } \overline{\text{AF}} = \overline{\text{Sin.}^2 \phi \times \text{Cos.} 2 \pi} \text{ and}$$

$$\overline{\text{BF}} = \overline{\text{Sin.}^2 \theta \times \text{Cos.} 2 \omega}, \text{ and therefore}$$

$$\overline{\text{AF} + \text{BF}} = \overline{\text{AB}} = \overline{\text{Sin.}^2 \phi \times \text{Cos.} 2 \pi} + \overline{\text{Sin.}^2 \theta \times \text{Cos.} 2 \omega}$$

which is the very same value which we have found above for $\overline{\text{Cos.}^2 x}$, the measure of the tint produced by one axis.

Hence it follows that the intensity of the tints, and consequently the form of the curves of equal tint, are the same, whether they are produced by one negative, or by two equal and rectangular positive axes.

We may therefore conclude,

1st. That a single negative axis may be resolved into two

equal rectangular positive axes of the same intensity as the negative axis, and lying in a plane perpendicular to that axis.

This fundamental principle being established, we may proceed still farther in the resolution of the axes into axes of equal intensity, but differing in their character and position. Thus in Pl. xvi. fig. 9, the negative axis A being equivalent to the positive axes B,C, we may again resolve B into a negative axis at C and another at A. Hence the original axis at A is equivalent to a negative axis at A, a positive axis at C, and a negative axis at C. But these two last axes destroy each other; and therefore the negative axis A is the only one that is left. If we also resolve the other positive axes C into two negative axes at A and B, we have the original negative axis A, equivalent to four negative axes, two at A, one at C, and one at B. We therefore conclude,

2d. That the effect of a single negative axis may be represented by three rectangular negative axes, provided two of them are equal, and the third has a greater intensity than the other two; and *vice versa* a single positive axis may be represented by three rectangular positive axes, provided two of them are equal, and the third has a less intensity than the other two.

Hence it follows that by leaving one axis at A, the effect of the other three negative axes must be to destroy each other, a result which will afterwards be established in a different way, and will lead us to some important consequences. The same is true of positive axes, *mutatis mutandis*.

Hitherto we have resolved the axes of crystals into other axes differing in position and character, but having the same intensity. If we consider, however, that any one axis at A

is equal to any number of equal or unequal axes of the same name placed at A, the sum of whose intensities is equal to the intensity of the axis at A, it will follow—

3d. That the effect of a single axis may be represented by any even number of equal axes of an opposite name, provided in every pair of axes one of them is at right angles to the other, and all of them lie in the same plane perpendicular to the original axis. The intensity of each of the new axes must be to that of the single axis as $\frac{n}{2}$ to 1, n being the number of axes.

4th. That the effect of a single axis may be represented by any number of equal pairs of axes of an opposite nature, provided the axes which compose each pair are equal and rectangular, and all of them lie in the same plane perpendicular to the original axis. The intensity of the axes which compose each pair may differ in any way, but $\frac{s}{2}$ or half the sum s of the intensities of all the axes must be equal to the intensity of the single axis.

In the preceding observations we have supposed the axes of the crystals to be rectangular, a supposition which is by no means rendered necessary by the phenomena. It is obvious indeed that the effect of a single axis cannot be represented by two axes that have any other inclination but that of 0° or 90° ; but we shall presently see that the effect of two rectangular axes may be represented by two equal and inclined axes. Let ABC, Pl. xvi. fig. 10, be a spherical triangle representing the eighth part of the sphere, and having all its angles right angles. Then if G is the pole of one of the resultant axes or diameters of no polarisation, the angular distance $\alpha \beta$ of the two inclined axes α, β must be such, that the angle $\alpha G \beta$ is a

right angle, for it is only at this angle that the two axes can destroy each others' actions, and produce a resultant axis at G. Consequently, since α GF is 45° , we have $\text{Tang. } \frac{1}{2} \alpha\beta = \text{Sin. } GF = \text{Cos. } AG$. The same reasoning is applicable if the axes are situated at a, b , a b being a right angle; but in this case $\text{Tang. } \frac{1}{2} ab = \text{Sin. } AG$. Hence,

5th. The action of two unequal rectangular axes is equal to the action of two equal axes, the tangent of the half of whose inclination is equal to the cosine of half the angle of the resultant axes, if the two equal axes are situated in the plane BC, or to the sine of half that angle if they are situated in ab .

In order to ascertain the intensity of the axes α, β , or a, b , we must assume a position and a character for the rectangular axes which they represent. Let the two rectangular axes, therefore, be a negative axis at A, and a positive axis at F, whose relative intensities will be $\text{Cos.}^2 GA : \text{Sin.}^2 GA$, and let A be the most powerful axis. Then the intensity of either of the axes α, β will be to that of A as $\frac{1}{2 \text{Sin.}^2 \frac{1}{2} \alpha\beta} : 1$; and the intensity of either of the axes a, b will be to that of A as $\frac{1}{2 \text{Cos.}^2 \frac{1}{2} ab} : 1$. Hence when $\alpha\beta = 90^\circ$. we have $\frac{1}{2 \text{Sin.}^2 \frac{1}{2} \alpha\beta} = 1$, which brings us back to Case 1st. When $ab = 0^\circ$. we have

$\frac{1}{2 \text{Cos.}^2 \frac{1}{2} ab} = \frac{1}{2}$, for as the two axes a, b coincide with A, their sum must be equal to A, or $a + b = 1$, and since $a = b$, we have $a = \frac{1}{2}$. When B is the most powerful axis, the preceding ratios of the intensities must be interchanged.

If the two inclined axes α, β are supposed to be unequal, they may have an infinite number of positions in the great circle passing through α, β ; but their relative position must be such, that the great circles passing through each of them,

and the resultant axis must intersect each other at an angle of 90° , and their intensities will be as the squares of the sines of their respective distances from the resultant axis. The ratio of the intensities, however, can never exceed that of 1 to $\text{Sin.}^2 \phi$, ϕ being the distance of the resultant axis from the plane passing through α, β .

It would be needless to pursue this subject any farther. I have briefly illustrated the general principles of the resolution of the axes of crystals, and the reader will have no difficulty in deducing other combinations by which the phenomena may be represented. A very important question is naturally suggested by the results to which we have arrived. Are there any physical circumstances either of a general or a particular nature which may lead us to ascertain the real position of the axes of crystals, and to determine the character of the forces by which the phenomena of polarisation are produced? When we consider the case of *Iceland spar*, we perceive no peculiarities which can induce us to refer its polarising force to two or more positive axes in preference to a negative axis; but in *rock crystal*, the secondary tints discovered by M. BIOT along the axes of the prism, seem to indicate that its apparent positive axis is merely the resultant of two or more equal and rectangular negative axes. M. BIOT indeed ascribes these secondary effects to new forces independent of the principal polarising force; but I have discovered them also in crystals with two axes, and have observed some phenomena which seem to prove that they have their origin in the unbalanced action of the two principal axes.

With regard to the nature of the forces we are not left

entirely without some general indications. In magnetism and electricity, the various phenomena are produced by two opposite and co-existent forces which modify each other's action ; and since opposite forces are obviously indicated by the phenomena of polarisation, we have the strongest reasons, from analogy, to believe that they are also co-existent in crystals. The remarkable phenomena exhibited during the transmission of heat along plates of glass, give additional weight to the deductions of analogy. Here the negative structure invariably accompanies the positive structure, and the plates which are under the influence of these forces, exhibit phenomena precisely the same as artificial magnets.* It will, however, be naturally asked, what has become of the negative force in calcareous spar, if it is supposed to have two equal and rectangular positive axes ? To this we reply, that the negative force, originally less than the positive forces, has been balanced by the opposite actions of the two positive axes ; and that the resulting force, which has all the characters of a negative force, is merely the difference between the original negative force and the negative force which is equivalent to the effect of the positive axes. Nor is this mere speculation, for while it is an arrangement of the polarising forces which is as likely as any other to be the one actually followed by nature, it is almost directly supported by a series of experiments, which I have made on plates and cylinders of glass.†

* *Phil. Trans.* 1816, p. 64, 83, 84, &c.

† See the *Edinburgh Transactions*, vol. VIII. p. 353.

SECT. V. *On the polarising structure of crystals that have the cube, the regular octohedron, and the rhomboidal dodecahedron for their primitive form.*

In examining the structure of doubly refracting crystals, it will be found that there are *thirteen* bodies which seem to be entirely destitute of the polarising structure, and other *nine* which sometimes exhibit distinct traces of two opposite structures.* And it is a very curious circumstance, that the crystals which have this character are those which crystallize in the form of a cube, a regular octohedron, and a rhomboidal dodecahedron, as appears from the following table.

<i>Names of crystals.</i>	<i>Primitive form.</i>	<i>Names of crystals. Primitive form.</i>
Garnet	Rhomboidal dodecahedron	Boracite†
Blende	Rhomboidal dodecahedron	Muriate of soda
Diamond	Regular octohedron	Muriate of potash
Spinelle ruby	Ditto	Leucite
Ceylanite	Ditto	Analcime
Alum	Ditto	Nitrate of barytes
Muriate of ammonia	Ditto	Sulphate of alumine and ammonia
Fluor spar	Ditto	Nitrite of lead
Ruby copper	Ditto	Cinnamon stone
Nitrate of lead	Ditto	Essonite
Nitrate of strontian	Octohedral crystals	Uranite

These crystals, as will be seen by a comparison with HAÜY's table of primitive forms,‡ include all the transparent and translucent crystals which he has arranged under the preceding forms, excepting *tungstate of lime*; but this ex-

* See the *Edinburgh Transactions*, vol. viii. p. 157, where I have given an account of the discovery of this property.

† Boracite has an axis in every direction, like the waxen partitions of the honeycomb.

‡ *Traité de Mineralogie*, tom. i. p. 273.

ception is removed, as he has since found that this mineral has for its primitive form an irregular octohedron.

In the paper already quoted, I have described the leading optical properties of this class of crystals; but I have since observed several new phenomena which throw additional light on the subject. In the examination of thirty very fine cut diamonds of great size and value, I found several which had no action whatever upon light. Many of them polarised a fine sky blue tint of the first order, with a complementary straw yellow, in whatever position they were held; and one exhibited a succession of five or six tints, which were not the exact tints of NEWTON's scale, but similar to those seen near the resultant axes of crystals. In a diamond with natural faces, which has the form of a fine regular octohedron, there were no indications whatever of the polarising structure. In two specimens of *leucite*, broad segments of the coloured rings were developed when the light was transmitted in different directions; and *blende* and *analcime*, even at thicknesses less than $\frac{1}{25}$ of an inch, displayed a considerable action upon polarised light.

M. BIOT has endeavoured to account for the absence of polarisation and double refraction in this class of crystals by remarking—" that the phenomena of the attractive class are explicable by a prolate ellipsoid and those of the repulsive class, by an oblate ellipsoid; and that the sphere, forming a passage from one of these limits to the other constitutes a sort of neutral state, and corresponds to those crystals which, crystallizing in the regular octohedron or the cube, are destitute of the property of double refraction."*

* *Traité de Physique*, tom. iv. p. 349, Paris, 1816.

This remark, however, is by no means an explanation of the fact. It is merely a different mode of expressing the fact, that the cubical and octohedral crystals have no double refraction; for the sphere is applicable to the optical action of air and all other fluids, whether aeriform or liquid, which have not the property of double refraction.

The discovery of the remains of polarising axes in a great number of crystals of this class, completely proves that, like all other crystallized bodies, they actually possess the doubly refracting and polarising structure, but that this structure entirely vanishes in some specimens, by the equilibrium of the forces in every point of the crystal, and reappears in some specimens when that equilibrium is not complete.

The principles of the resolution of polarising forces which we have already explained, indicate the manner in which this equilibrium is effected. Three equal and rectangular axes, either all of the negative, or all of the positive character, will produce a perfect equilibrium, or mutually destroy each other at every point of the crystal; for since two equal positive axes are equivalent to one negative axis, of the same intensity as either of the two; and since this negative axis will be balanced by an equal positive axis coincident with it, it follows, that the three rectangular positive axes will be in equilibrio. In perfect crystals of this class where all the axes are equal, and their position exactly rectangular, the equilibrium of the forces is complete; but if the axes are not equal, or their position not accurately rectangular, the phenomena of one or more axes will be developed. If one axis, for example, is weaker than the other, then the result will be the appearance

of an axis of an opposite name to that of the three axes and coinciding with the resultant of the other two; but if one of the axes is stronger than the other two, the result will be an axis of the same name with the three axes, and coinciding with the strongest axis.

Let us now consider what will be the effect if the three negative axes C,A,O, Pl. xv. fig. 4, are all different; C being the strongest and O the weakest. The two axes C and A will, by their joint action, produce resultant axes P,P' lying in the plane COD. Let the third axis O be resolved into two positive axes at C and A, then we have a negative axis A, and a positive axis equal to O acting at A, and the negative axis C, and a positive axis equal to O acting at C; but O is less than A or C, therefore we have a negative axis equal to A—O acting at A, and a negative axis C—O acting at C; and as the last of these is the strongest, the general effect will be the production of two resultant axes in the plane COD. But as C—O has to A—O a greater ratio than C has to A, the poles of no polarisation P,P' will be removed farther from O than they were by the action of C and A alone.

The equilibrium of the three axes may also be disturbed by another cause, namely, by a deviation from perfect rectangularity. In order to understand the effect of this irregularity, let ABC, Pl. xvi. fig. 10, be the three rectangular axes, and B α , C β the deviation of the axes α and β . Then the effect of the equal and inclined axes α , β is to produce poles of no polarisation, one of which is seen at G in the plane FGA, bisecting the angle formed by the planes passing

through the axes. If the third axis is rectangular to α and β , or placed at A, the effect of it would be to throw the pole G farther from A towards F; but as it is not probable that the deviation of α and β would be in a plane exactly at right angles to A, while A kept its situation; then if A deviates from A, or if while A retains its position, the deviations of α and β are not in the plane BC, an irregular system of curves will be produced, differing essentially from those which are formed by two axes.

Here then we have three leading causes of the destruction of the equilibrium of the three axes. In consequence of the first, tints of a single structure will be developed, as in some specimens of diamond and alum. In consequence of the second, the opposite tints of a double structure will be developed, as in several specimens of diamond, fluor spar, muriate of soda, semiopal, and analcime; and in consequence of the third, a confused system of rings will be exhibited, such as I have discovered in several specimens of leucite.

The coincidence of the preceding deductions, with the experimental results, affords a strong presumption, that we have been tracing the actual operations of nature. In the phenomena of crystals both with one and with two axes, we have seen that these axes are coincident with some permanent line in the primitive or secondary form of the crystal. This is the case in every crystal with one axis that I have examined; but it is not easy to establish this coincidence in crystals with two axes, for it is impossible to fix the situation of the axes by which the phenomena are produced. In almost every case, however, the most probable position of the axes is

coincident with some prominent line, either in the primitive or secondary forms.*

The same coincidence ought therefore to be expected in the case of the cubical, octohedral, and rhomboido-dodecahedral crystals ; and it is a singular fact, that the *cube*, the *octohedron*, and the *rhomboidal dodecahedron*, are the only regular geometrical solids in crystallography, in which neither more nor less than three equal rectangular axes can be placed symmetrically. In the *cube*, for example, each of three axes is perpendicular to the three pair of square surfaces by which the solid is contained. In the *regular octohedron* each of them coincides with the line which joins the six solid angles of the figure ; and in the *rhomboidal dodecahedron*, each of the rectangular axes passes through the six solid angles, each of which is contained by four acute angles of the rhomboidal planes.

SECT. VI. *On the artificial imitation of all the classes of doubly refracting crystals by means of plates of glass.*

In the Philosophical Transactions for 1816, I have given a full account of the very remarkable phenomena which are

* Malus seems to have believed that the axis of extraordinary refraction was necessarily coincident with some prominent line in the primitive form. We quote the following passage in confirmation of our general views, though there can be no doubt that the generalisation which it implies is premature. “*Dans le rhomboïde, l'axe de réfraction se confond avec l'axe du cristal ; mais dans les autres formes on n'a pas de données suffisantes pour le déterminer à priori. Cependant le nombre des directions entre lesquelles on peut balancer est toujours très-borné. Dans l'octaèdre à triangle scalène, par exemple, on est assuré d'avance que l'axe de réfraction est un des trois axes rectangulaires de la forme primitive.*” *Théorie de la Double Réfraction*, p. 177.

exhibited during the propagation of heat along plates of glass and other uncryallized bodies; and of the method of communicating permanently to these substances, all the properties of regular crystals. Since that paper was written, I have completed a series of new experiments on the distribution of the polarising force in prepared glass, which led me to the determination of the laws by which all the phenomena can be calculated. These laws are precisely the same as those which we have already deduced in the preceding sections; and there is not one phenomenon belonging to regular crystals which I have not been able to imitate in glass.

In these artificial crystals, however, the centre of the sphere is infinitely distant, and therefore the sine of the angle which the refracted ray makes with the axis in the regular crystals, is equivalent in the artificial ones to the distance of the ray from the axis, and is replaced by this quantity in the new formulæ. Thus if T is the tint corresponding to any given distance D from the axis, then the tint t at any other distance, d is $t = \frac{T d^2}{D^2}$ a formula which represents accurately the progress of the tints whether the axis is negative or positive.

If in rectangular plates of glass D represents the distance of any of the black fringes, or the lines of no polarisation MN, OP , Pl. xvi. fig. 13, from the axis ab of the plate, and T the maximum central tint; then if we conceive an axis situated perpendicular to the plate in every point of the line ab , and an axis in the plane of the plate perpendicular to the former axis, T will be the tint produced in every part of the plate by the axis in its plane when polarised light is transmitted through it perpendicularly. But as this tint, whose value is

T , is counteracted at m in every part of the line MN , by the action of the axes at A , &c. perpendicular to the plate, the intensity of the perpendicular axis at the distance $A m = D$, will also be equal to T . Hence, in order to find the tint t produced by this perpendicular axis at any other distance $A c = d$, we have $t = \frac{T d^2}{D^2}$ by the law for one axis; but as this tint is counteracted by the tint T produced at c by the axis in the plane of the laminæ, we have the resulting tint or $t = T - \frac{T d^2}{D^2}$.* Here the law is very simple, and the tints are all disposed in straight lines, in consequence of the angle of the forces being 180° , and the number of the axes infinite. This case is precisely the same as that of regular crystals when the tints are calculated in the plane COD , Pl. xv. fig. 4, by means of two axes at O and A , the effect of the axis A is the same at every point in the line COD , and therefore the resulting tint, at any point, is the uniform tint produced by A (which is the same as the tint at O) minus the tint produced by the axis O alone. The law of the progression of the tints therefore in rectangular plates of glass, is exactly the same as in crystals with two axes; and we have the same difficulty in determining whether the axes are of the same or of opposite names.

The similarity between the various phenomena of the real

* This formula is the same as that which I had deduced long ago from experiment. M. BIOT had also obtained from observation the very same expression of the tints, and about the same time. The formula which I have employed was more general than that of M. BIOT, as I had found the term D to be a function of the breadth of the plate. My formula was therefore $t = T - \frac{T d^2}{312 B^2}$, B being the breadth of the plate; or if T is the maximum tint given by the plate, we have $t = T - \frac{T d^2}{188 B^2}$. See *Edinburgh Transactions*, Vol. VIII. Art. XVIII.

and the artificial crystals is still more striking than the agreement of the laws of the tints, as all the different classes of crystals can be imitated in glass. This similarity will appear from the following comparison :

<i>Artificial crystals.</i>	<i>Real crystals.</i>
1 A circular plate of glass crystallized by the transmission of heat.	1 Crystals of zircon, quartz, ice, &c.
2 A circular plate of glass heated, and in the act of cooling.	2 Crystals of calcareous spar, beryl, ruby, &c.
3 A rectangular plate of glass when heat is in the act of being transmitted through the plate.	3 Crystals of topaz, sulphate of lime, &c. in which the principal force is attractive.
4 A rectangular plate of glass heated, uniformly, and in the act of cooling.	4 Crystals of mica, nitre, &c. in which the principal force is repulsive.
5 A circular plate of glass, permanently crystallized to a slight degree, and then heated, so as to produce, in the act of cooling, a tint equal to its permanent tint.	5 Cubical, octohedral, and rhomboido-dodecahedral crystals, in which the polarising force is destroyed by opposite actions.

Although these artificial crystals exhibit in the most perfect manner all the phenomena of fixed and moveable polarisation, and develope the tints of NEWTON's scale with much more regularity than even sulphate of lime, yet M. BIOT has described the phenomena which they exhibit under the head of *imperfect crystallization*. If such a distinction were neces-

sary for the proper classification of the phenomena, I would have no hesitation in reversing the name, and giving the appellation of imperfect crystallization to the phenomena produced by *sulphate of lime*, and the greater number of crystals of the mineral kingdom. If we employ pieces of pure glass of regular forms, and with straight and smooth edges, and if we transmit through them the heat in a uniform and careful manner, we shall obtain a structure which will develope all the phenomena of polarisation with the utmost beauty and perfection.

There is one point, however, and a very instructive one, in which the artificial differ from the real crystals. If we cut a plate of calcareous spar into any number of pieces, each piece, however minute, will produce the system of coloured rings in as great perfection as the whole plate of which it formed a part ; and its polarising force will suffer no diminution by the reduction of its size, provided its thickness remain the same. If we subdivide, however, a circular plate of glass, the polarising force of each portion is not only much diminished, and sometimes even destroyed, if the portion is very small, but the polarising influence is distributed in a new manner, according to the outward shape of the fragment.* Hence it follows, that in glass the polarising structure *depends entirely on the external form of the plate*, and on the mode of aggregation of its particles. When its form is circular, it has only one axis of polarisation ; and this axis is positive, if the density diminishes towards the centre, and negative, if it increases towards the centre ; but when

* See *Phil. Trans.* 1816, pp. 71, 82.

its form is rectangular, or elliptical, it has two axes of polarisation, the strongest of which appears to be positive, and the weakest negative.

In crystals, on the contrary, the polarising structure is wholly independent of their outward form, and of any variation of density in the aggregation of their particles, and must therefore depend on the form of their integrant molecules, and the variation in their density. Is it not more than probable, therefore, that we have in our artificial crystals, a representation of the ultimate particles of crystallized bodies? When these crystals have a spherical form, diminishing in density towards an axis, and have these axes arranged by the laws of crystallization, they will constitute a crystal of the positive class, like *ice*, *zircon*, *quartz*, &c.; and the only difference between ice and water will be, that in the ice the particles have their axes regularly disposed; while in the water, the axes have every possible direction, so as to create a general equilibrium of the polarising forces. When the density of the spheres increases towards their axes, their symmetrical combination will constitute a crystal of the negative class, such as *beryl*, *calcareous spar*, &c.

But if the particles, instead of being spheres, are either prolate or oblate spheroids, having their polarising axis at right angles to the axis of revolution; and if their density varies, as in elliptical plates of glass, along which heat is in the act of being transmitted, they will constitute by their regular combination, the class of crystals with two axes, such as *topaz*, *sulphate of lime*, &c. in which the positive force predominates. If the density, on the other hand, varies as in

elliptical plates heated in oil, and in the act of sudden cooling, the combined spheroids will constitute the class of crystals like *mica*, *nitre*, &c. in which the negative force predominates.

The variation of density which these solids necessarily require, in order to develope a force varying as the fourth power, or an influence upon the tints varying as the square of the sine of the angle which the ray forms with the axes of the crystals, will be understood from Pl. xvi. fig. 14. Let $M a N b$ represent the section of the sphere through one of its great circles, then if we suppose that the density is a maximum in every point of the line MN , and that it diminishes from M to ab , and from L to dc , at first very slowly, and then very rapidly, like the cosines of the angles reckoned from MP , so that the lines of equal density in every section of the sphere passing through MN are diameters, the sphere will represent one of the integrant particles of the positive crystals, like *ice*, *zircon*, *quartz*, &c. having MN for its axis of double refraction and polarisation. In like manner, if the density is a minimum in the line MN , and increases in a similar manner, towards ab and cd , the sphere will represent an integrant particle of the negative crystals, such as *beryl*, *calcareous spar*, &c. When the polarised ray is transmitted along the axis MN , the polarising force will be nothing; when it passes along RS , the variation of density is greater, and consequently the polarising force will be increased; and when it is transmitted along ab , the polarising force will be a maximum, as the variation of the density is a maximum in that line. As the same is true of every other section of the sphere passing through MN , it follows that there will be a

certain rate of variation in the density at which the polarising force will be proportional to the fourth power, or the variable influence upon the tints to the square of the sine of the angle which the polarised ray forms with the axis.

The preceding reasoning must not be considered as mere speculation, for such a crystal as that represented in fig. 14, may be actually constructed with plates of glass. Let ABCD, for example, represent in section, a circular plate of glass whose thickness is ML, and along whose axis MN, is seen the system of coloured rings shown in Pl. xv. fig. 1. Then, if we conceive the plate bent back into the position $a M b c L d$, we have the external spherical stratum of our elementary sphere; and in like manner we may conceive the interior strata to be formed by a succession of circular plates DEFC, &c. bent into the spherical form $d L c f K e$. Now, since the tints of the circular plate ABCD vary as the squares of the distances from its axis ML, we may suppose that the same law still exists, after it is bent into $a M b c L d$. But the distances from the axis MN are now the sines of the angular distances from M; and therefore, since the same is true of all the other spherical strata, of which the elementary sphere is composed, it follows that the tints produced by the transmission of polarised light, along any diameter of the sphere, are proportional to the square of the sine of the angle, which the ray forms with the axis of the crystal. In order to construct this sphere artificially, we have only to crystallize a series of hemispherical strata, and join them together at the line ab in the manner represented in the figure. This sphere, it will be readily seen, is totally different from a solid sphere of crystallized glass, which has no particular axis, but which gives

the same phenomena by transmitting polarised light through any of its diameters.*

In the very same manner we may suppose the elementary spheroid of crystals with two axes to be formed by elliptical plates, bent into spheroidal strata, and the spheroid itself may be actually constructed by means of spheroidal strata of glass. When this is done, it will exhibit all the complicated phenomena produced by the simultaneous actions of two unequal axes.

SECTION VII. *On the laws of double refraction in crystals with any number of axes.*

From the coincidence which I have invariably observed between the axes of polarisation and extraordinary refraction, in all crystals that have only one apparent axis, we may consider it as an established fact, that the HUYGENIAN law represents, within the limits of experimental errors, not only the phenomena of calcareous spar, but those of all other crystals with a single axis of double refraction.

But since it is the general character of crystals to have more than one axis of polarisation, it becomes interesting to ascertain, if they also have two axes of extraordinary refraction; and if they have, to investigate, by direct experiment, the general laws of double refraction, for crystals with any number of axes. Such an investigation presents difficulties of no ordinary kind. The want of a transparent mineral with two powerful axes, which, like calcareous spar, could be obtained in large pieces, and cut with facility in every direc-

* See *Phil. Trans.* 1816, pp. 314, 315.

tion ; and the necessity that it should have its resultant axes considerably inclined to each other, in order to obtain a measurable separation of the images at several points, between these resultant axes, rendered all my experiments for a long time completely unsuccessful. The discovery, however, of crystals which possessed, in some degree, the most important of these requisites, has enabled me to resume and to complete the investigation.

After ascertaining the position of the diameters of no polarisation in a crystal with two axes, I formed a prism with a great refracting angle, so as to have a flat surface as perpendicular as possible to one of these diameters. I then placed this prism upon the goniometer, and having marked the position when the point of no polarisation, or the centre of the rings (namely P, Plate xv. fig. 4.) was coincident with the polarised image of a candle, I substituted the direct image of a candle, and observed that this image was single, and therefore that the force of double refraction, as well as the polarising force, had completely vanished. Upon turning the goniometer to both sides of these positions, the deviation of the extraordinary ray became perceptible and gradually increased ; and in continuing to observe this deviation in the plane COD passing through the resultant axes, I found that it increased to the middle point O between these axes, where it became stationary ; and that it again gradually diminished towards the other resultant axis P' where the image again became single. I now measured the deviation of the extraordinary ray at the points O, A, C, and at various other points, both in and out of these rectangular directions, and I found that the force of double

refraction, varied in the same ratio as the polarising force, and that all the phenomena, whatever be the number of axes by which they are produced, may be calculated by the same general law which we have already established for the phenomena of polarisation.

Let it be required, for example, to determine the velocity of the extraordinary ray in a crystal with any number of positive and negative axes. By the principles explained in Sect. IV. these axes may be reduced to two equivalent rectangular axes, which may be either of the same, or of opposite names. Let us then take b = the axis of revolution of the two spheroids, a, a' the other axis, β, β' the inclination of the incident ray to the axes of the crystal, ψ the angle of the forces as found in Sect. III. and ζ half the difference of the angles at the base of the parallelogram of forces. Then since the velocity of the light is inversely as the variable radius of the spheroid $\frac{1}{b^2}$ will be the square of the velocity of the ordinary ray, and $\frac{1}{a^2} - \frac{1}{a'^2}$ the square of the minimum velocity of the extraordinary ray in virtue of the separate action of each axis. The difference between the squares of the velocities of the ordinary and extraordinary rays will be

$$\left(\frac{a^2 \pm b^2}{a^2 b^2} \right) \sin^2 \beta$$

$$\left(\frac{a'^2 \pm b^2}{a'^2 b^2} \right) \sin^2 \beta'$$

the sign being positive, when the axis is positive, and vice versa. But as these expressions represent the sides of the parallelogram of forces, we have

$$\text{Tang. } \zeta = \frac{\left(\frac{a^2 \pm b^2}{a^2 b^2} \sin^2 \beta - \left(\frac{a'^2 \pm b^2}{a'^2 b^2} \sin^2 \beta' \right) \tan \frac{1}{2} \psi \right)}{\left(\frac{a^2 \pm b^2}{a^2 b^2} \sin^2 \beta + \frac{a'^2 \pm b^2}{a'^2 b^2} \sin^2 \beta' \right)}$$

Consequently the difference between the squares of the velocities of the ordinary and extraordinary ray produced by the combined action of the two axes will be

$$\frac{\left(\frac{a^2 \pm b^2}{a^2 b^2} \sin.^2 \beta\right) (\sin. \psi)}{\sin. (\zeta + \frac{1}{2} \psi)}$$

Hence, calling V the velocity required, we have

$$V^2 = \frac{\frac{1}{b^2} \pm \left(\frac{a^2 \pm b^2}{a^2 b^2} \sin.^2 \beta\right) (\sin. \psi)}{\sin. (\zeta + \frac{1}{2} \psi)} \text{ and}$$

$$V = \left(\frac{\frac{1}{b^2} \pm \left(\frac{a^2 \pm b^2}{a^2 b^2} \sin.^2 \beta\right) (\sin. \psi)}{\sin. (\zeta + \frac{1}{2} \psi)} \right)^{\frac{1}{2}}$$

The form of the compound, or irregular spheroid, may therefore be computed for all doubly refracting crystals.

The general law of double refraction which has now been explained, may be thus expressed.

The increment of the square of the velocity of the extraordinary ray produced by the action of two axes of double refraction, is equal to the diagonal of a parallelogram whose sides are the increments of the square of the velocity produced by each axis separately, and calculated by the law of HUYGENS, and whose angle is double of the angle formed by the two planes passing through the ray and the respective axes.

When the two axes are of equal intensity and of the same character, the preceding law gives the very same results as the law of HUYGENS does for one axis placed at right angles to the other two.

It is scarcely necessary to observe, to those who have studied the preceding sections, that the phenomena of double refraction cannot be referred to the *ordinary* action of at-

tractive and repulsive forces. If a ray of light is exposed to the ordinary action of a repulsive or an attractive force emanating from two equal and rectangular axes, there is no point of incidence at which the resultant of these forces is nothing, or there is no resultant axis along which the ray ceases to be divided. In like manner, if the ray is exposed to the action of two equal and rectangular axes, one of which is repulsive and the other attractive, there are innumerable points of incidence in which the resultant of the forces is nothing, and these points are situated in the circumference of two great circles perpendicular to the plane passing through the axes, and bisecting the right angle which they form with each other. These, however, are results entirely incompatible with the actual phenomena of double refraction.

In the preceding sections of this letter, I have included only the more general results of my researches, and have reserved, for another paper, an account of my experiments on the absolute polarising forces of crystals; on the form of the ellipsoids by which their double refraction is regulated, and on the position of their axes with respect to their natural faces, or with respect to certain fixed lines in their primitive forms.

In treating of the coloured rings produced by polarised light, I have likewise omitted the phenomena which are peculiar to individual crystals; and I have not ventured to adopt any theory of their formation.

I cannot, however, conclude this paper without noticing the happy application which Dr. THOMAS YOUNG has made of his beautiful law of interference to the explanation of

this class of colours. The various phenomena of thick and thin plates ; the colours of inflected light ; the hues of the supernumerary rainbow ; the fringes which I observed with two inclined plates of glass of equal thickness ; the colours noticed by Mr. NICHOLSON and Mr. KNOX, with plates of unequal thickness ; the communicable colours of mother of pearl and striated surfaces ; and the colours which I have discovered by the successive reflection of polarised light between metallic plates, and surfaces that produce total reflection, are all referable to this simple law. But while we thus admire the wide range of phenomena which the law of interference embraces, it is necessary also to state, that its application to the colours of polarised light is still attended with some difficulties, and that there are other phenomena of complementary colours which I have lately observed, that resist this method of classification. These, however, are merely difficulties in the application of the law, and not objections to its generality ; and I have no doubt that Dr. YOUNG will succeed in referring them to the same cause, and will thus add to the honour which already belongs to him, of having generalised a long train of perplexing and important phenomena.*

I have the honour to be, &c. &c.

DAVID BREWSTER.

To the Right Hon.

Sir Joseph Banks, Bart. &c. &c. &c.

* Having communicated to Dr. YOUNG the contents of the preceding paper, he has requested me to subjoin to it the following letter :

“ My Dear Sir,

“ Your experiments, on the colours afforded by crystals having two optical axes, appear to establish a very important result in the theory of light ; for

Fig. 1.

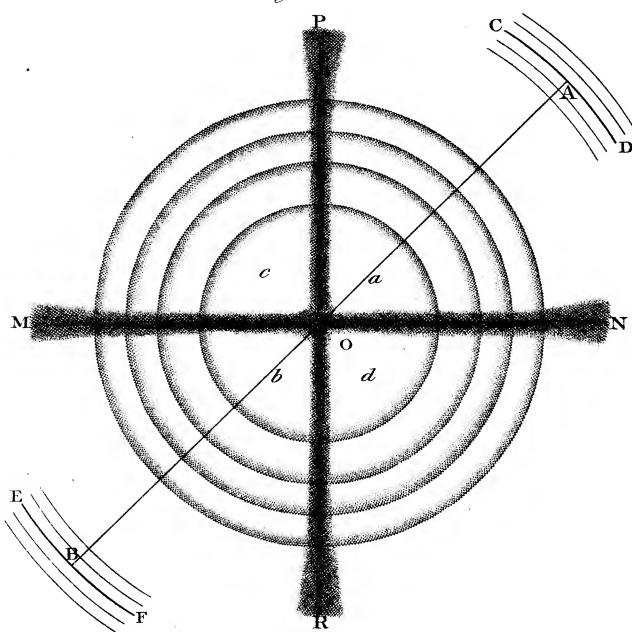


Fig. 2.

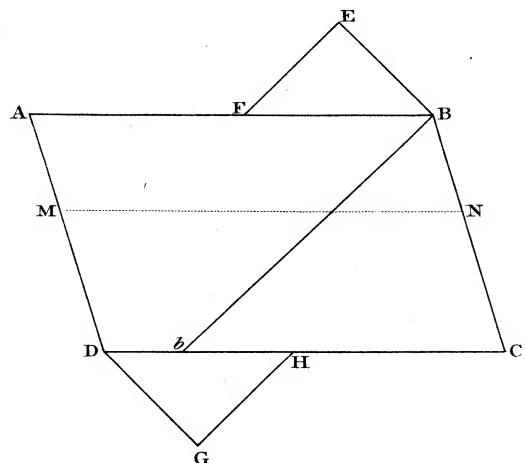


Fig. 3.

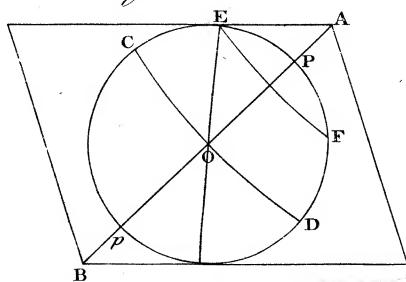


Fig. 6.

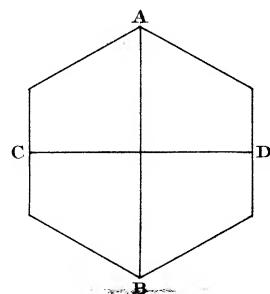


Fig. 4.

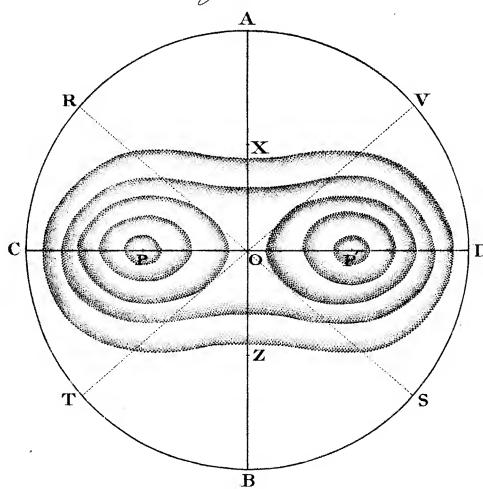


Fig. 5.

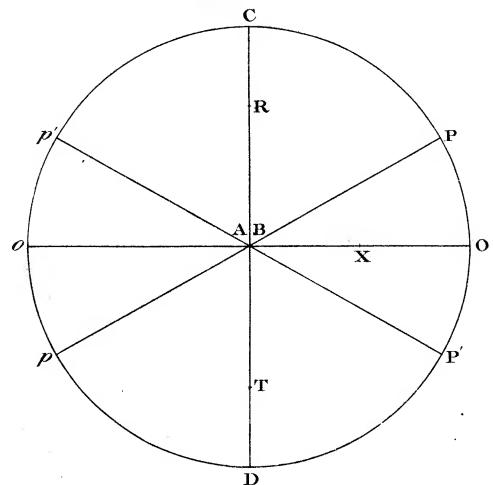


Fig. 7.

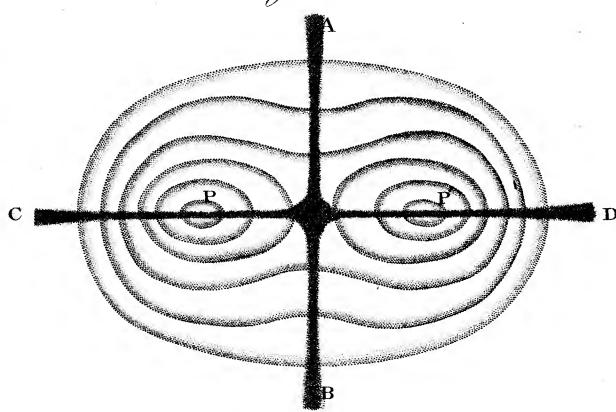


Fig. 8.

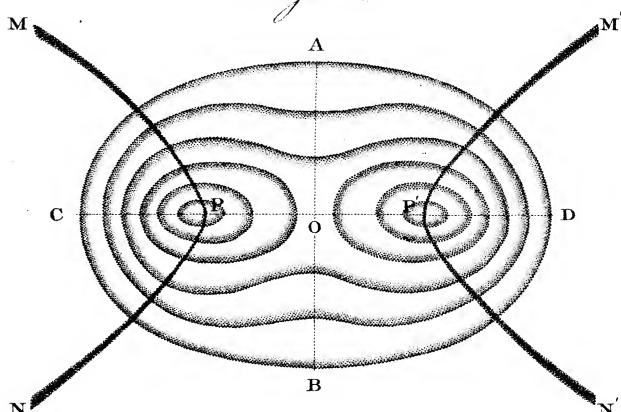


Fig. 9.

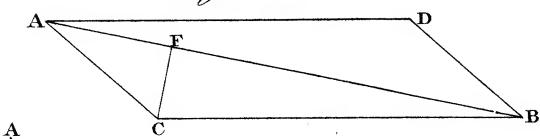
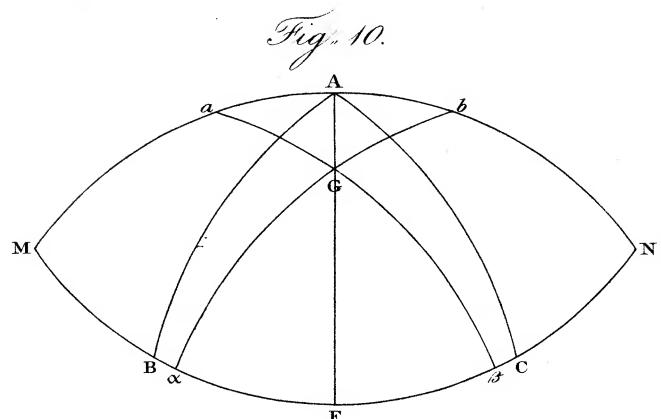
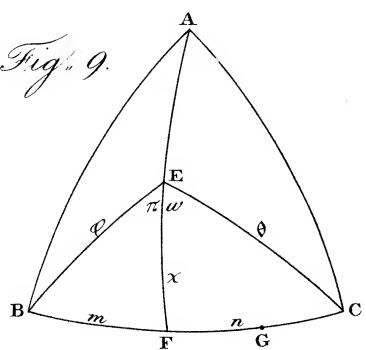


Fig. 11.

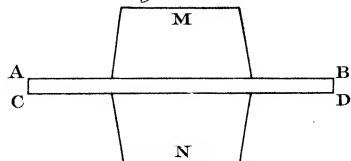


Fig. 14.

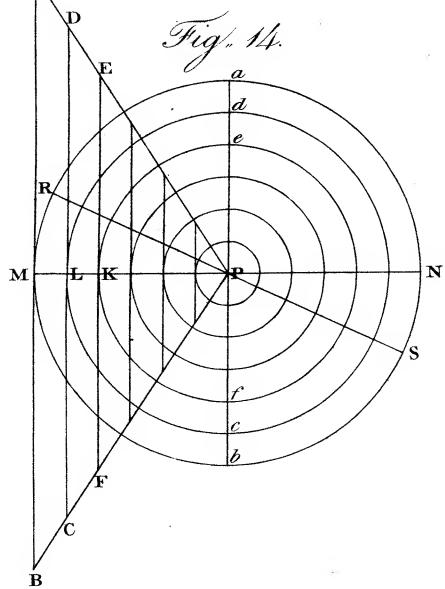
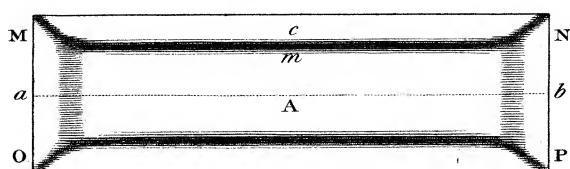


Fig. 13.



supposing them to be perfectly represented by your general law, it will follow that the tint exhibited depends not on the difference of refractive densities in the direction of the ray transmitted, but on the greatest difference of refractive densities in directions perpendicular to that of the ray. These two conditions lead to the same result, where the effect of one axis only is considered, but they vary materially where two axes are supposed to be combined; and I do not immediately perceive by what modification it will be possible to accommodate the laws of interference to these experiments. There can be little doubt that the direction of the polarisation, in such cases, must be determined by that of the greatest and least of the refractive densities in question †; and it seems to be very possible to apply your mode of calculation to many other phenomena, in which the polarising powers of different crystals are combined.

Believe me, dear Sir,

Your faithful and obedient servant,

THOMAS YOUNG."

† This supposition of Dr. YOUNG's is quite correct. In another paper, which will soon be submitted to the Royal Society, I have given a general method of finding the direction of the polarisation for any combination of axes.